AD NUMBER

AD859392

NEW LIMITATION CHANGE

TO

Approved for public release, distribution unlimited

FROM

Distribution authorized to U.S. Gov't. agencies and their contractors; Critical Technology; JUL 1969. Other requests shall be referred to Air Force Materials Lab., Attn: MAMD, Wright-Patterson AFB, OH 45433.

AUTHORITY

AFML ltr dtd 12 Jan 1972

AD-859392

AFML-TR-68-376

LENGTH OPTIMIZATION FOR CONSTRAINED VISCOELASTIC LAYER DAMPING

R. PLUNKETT

C. T. LEE

University of Minnesota

FILE COPY

TECHNICAL REPORT AFML-TR-68-376

JULY 1969

20070924068

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Metals and Ceramics Division (MAM), Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio 45433.

AIR FORCE MATERIALS LABORATORY
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention thay may in any way be related thereto.

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Metals and Ceramics Division (MAM), Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio 45433.

The distribution of this report is limited because protection of technical know-how relating to technology restricted by U.S. Export Control Act.

Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.

400 - August 1969 - CO455 - 88-1957

LENGTH OPTIMIZATION FOR CONSTRAINED VISCOELASTIC LAYER DAMPING

R. PLUNKETT

C. T. LEE

FOREWORD

This report was prepared by the University of Minnesota, Department of Aeronautics and Engineering Mechanics, under USAF Contract No. F33615-67-C-1667. This contract was initiated under Project No. 7351, "Metallic Materials," Task No. 735106, "Behavior of Metals." The work was administered under the direction of the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, with Mr. J. P. Henderson acting as project scientist.

This report covers work conducted from May 1967 through October 1968.

The manuscript was released by the authors December 1968 for publication.

This technical report has been reviewed and is approved.

W. J. Trapp

W. J. Trapp
Chief, Strength and Dynamics Branch
Metals and Ceramics Division
Air Force Materials Laboratory

ABSTRACT

Viscoelastic materials are extensively used to damp flexural vibrations of metallic structures; it has been known for some time that the energy dissipation due to shear strain in the viscoelastic layer can be increased by constraining it with a stiffer covering layer. In this report we will discuss a method for increasing this damping by cutting the constraining layer into appropriate lengths. The analysis for a single layer of this treatment is relatively straightforward. The damping can be increased still further by using several layers; in this case the analysis is based upon effective complex elastic moduli of an equivalent homogeneous medium. One result found from this analysis is that, for optimum spacing of cuts, the damping depends primarily upon the stiffness of the constraining layer and only slightly on the shear modulus of the viscoelastic layer. Experimental data is presented for comparison with the theoretical predictions.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

B_o^* = Characteristic length in single layer $= (t_1 \cdot t_2 \frac{E_2}{G_1^*})^{\frac{1}{2}}$ (in)

C^{*2} = G_E^*/E_E^*

d = Thickness of the beam (in.)

E_2 = Modulus of elasticity of the constraining material (psi)

E_b = Modulus of elasticity of the basic structure (psi)

E_E^* = Effective elastic modulus of the equivalent homogeneous material (psi)

f = Natural frequency $= \frac{\omega}{2\pi}$ (cycle/sec)

G_1^* = Complex shear modulus of the viscoelastis material (psi)

G_1 = $|G_1^*|$ absolute value of the shear modulus (psi)

G_1' = Storage shear modulus (psi)

G_1'' = Loss shear modulus (psi)

G_E^* = Effective shear modulus of the equivalent homogeneous material (psi)

H = Total thickness of the surface treatment = $m(t_1+t_2)$ (in.)

i = $\sqrt{-1}$

I = Moment of inertia of the basic structure (in.^{4})

ℓ = Length of the beam (in.)

L = Total length of the surface treatment (in.)

L_o^* = Characteristic length in multiple layer treatment (in.)

L_1 = Element length of the constraining layer

m = Number of layers of the surface treatment (integer)

n = Number of elements of the constraining layer = L/L_1

t = Time (sec)

t_1 = Thickness of the viscoelastic layer (in.)

t_2 = Thickness of the constraining layer (in.)

u^* = Axial displacement in the constraining layer (in.)

u = $|u^*|$ absolute value of the displacement (in.)

u_o = Displacement at the interface of the basic structure and the viscoelastic layer in the x-direction (in.)

ω = Dimensionless length ratio

w^* = Strain energy in the bare specimen (in.lb.)

$(W)_B$ = Energy stored in the bare specimen (in.lb.)

$(W)_S$ = Energy stored in a system (in.lb.)

$W_{NOM.}$ = Nominal energy stored in a material (in.lb.)

ΔW = Damping energy dissipation in a material (in.lb./cycle)

$(\Delta W)_B$ = Energy dissipation in the bare specimen (in.lb./cycle)

$(\Delta W)_L$ = Energy dissipation in the constrained viscoelastic layers (in.lb./cycle)

$(\Delta W)_S$ = Energy dissipation per cycle of a system (in.lb./cycle)

α^* = $L_1/2L_o^*$

β^* = Dimensionless ratio

γ^* = Shear strain in the viscoelastic layer

γ = $|\gamma^*|$ absolute value of the shear strain

γ_E^* = Effective shear strain of the equivalent homogeneous material

δ = Logarithmic decrement

ϵ_o = Uniform strain at the interface

ϵ_E^* = Effective normal strain of the equivalent homogeneous material

ζ = Damping ratio = $\frac{\log_e 10}{40\pi f} \cdot \frac{d(dB)}{dt}$

η_1 = Dimensionless loss coefficient

η_B = Loss coefficient of the bare specimen = $\frac{(\Delta W)_B}{2\pi (W)_B}$

η_G = Loss tangent of the viscoelastic material = G_1''/G_1'

η_L = Modified loss coefficient of the system = $\frac{(\Delta W)_L}{2\pi (W)_S}$

η_S = Loss coefficient of a system = $\frac{(\Delta W)_S}{2\pi (W)_S}$

θ = Loss angle of the viscoelastic material = $\tan^{-1} \eta_G$

ξ = Dimensionless length ratio = $\frac{L_1}{L_o}$

σ_1 = Normal stress in the viscoelastic layer (psi)

σ_2^* = Normal stress in the constraining layer (psi)

σ_2 = $|\sigma_2^*|$ absolute value of the normal stress

σ_E^* = Effective axial stress in the equivalent homogeneous material (psi)

τ^* = Shear stress in the viscoelastic layer (psi)

τ = $|\tau^*|$ absolute value of the shear stress (psi)

τ_E^* = Effective shear stress in the equivalent homogeneous material (psi)

$\bar{\chi}$ = Constant

ω = Natural frequency (rad./sec.)

I

INTRODUCTION

This report discusses the optimization of constrained viscoelastic layer damping for engineering structures such as beams, columns and plates. For such structures, the amount of damping for a given viscoelastic layer depends on the stiffness of the constraining layer. It also depends on an effective length for the constraining layer; this effective length may be related to the bending wave length, as Kerwin [1]* did, or it may be created by cutting the constraining layer at regular intervals as was shown by Parfitt [2]. Lazan et al [3] showed that the amount of damping can be increased by using alternately anchored multiple layer treatment; in this report we show that, when properly assembled, the constraining layers need not be anchored. An analysis, based on technical theory, for finite length and thickness of treatment is presented and the predictions of this theory are compared with experimental results for one to eight layers on a cantilever beam.

Viscoelastic damping layers can be used on the surface of structural members, so that under cyclic loading the viscoelastic layer experiences the cyclic extensional strains of the surface of the member [4,5]. In case of free viscoelastic layers, the shear strain and the dilatation in the viscoelastic layer are of the same order. If the viscoelastic layer is constrained by a stiff covering layer it experiences large shear strain and relatively small dilatation when the member to which it is attached is strained [5,6]. Since most of the energy dissipation is caused by shear deformation and almost none by dilatation [5], constrained viscoelastic layers are therefore capable of higher damping than unconstrained viscoelastic layers.

Kerwin [1] analyzed the damping of a composite structure with an infinitely long damping layer subjected to sinusoidal variation in bending moment. He found that the calculated damping factor depends on the wave length of bending waves in the damped structure as well as on the material properties and the geometry. Parfitt [2] determined the change in damping caused by cutting the damping tape at regular intervals; his analysis is valid only for materials with small loss coefficient since he neglected the difference between the absolute value of the shear modulus of the viscoelastic material and its real part. Lazan [3] gave an analysis of an alternately anchored multiple layer surface treatment which was developed for increasing damping.

In this report we consider the case of finite length surface treatment of an engineering structure with uniform surface strain and cyclic loading conditions. The constraining layer of the surface treatment is cut into appropriate lengths. If the constraining layer is very long, the shear stress near the ends induces the same

---

*Numbers in brackets designate reference at the end of this report.

axial strain in it as in the basic structure and thus there is no shear in the viscoelastic layer away from the ends and the damping is small. If the length of each element of the constraining layer is very short, it exerts no constraint on the underlying viscoelastic layer, there is little shear strain and the damping is small. At some finite value for the lengths of the elements of the constraining layer, the damping is a maximum.

An analysis based upon the technical theory of elasticity is developed in a straightforward manner for a viscoelastic layer constrained by a single stiff layer cut at appropriate intervals. For multiple layer surface treatment, there are interactions between the constraining layers and the viscoelastic layers on each side. The governing equations of equilibrium can still be written for each individual layer, but to solve this set of equations for a large number of layers would be very tedious. In order to simplify the analysis for the multiple-layered treatment, we replace a typical, repetitive, volume by an equivalent homogeneous material with the same force-deformation relationship. A longitudinal elastic modulus and a transverse shear modulus is found for this equivalent material in terms of the actual physical properties and geometry of the typical volume of the original composite. This equivalent analysis gives valid results if the composite has dimensions which are large in comparison with those of an element of the constraining layer and the strain in the basic structure does not vary too rapidly with length.

II

SINGLE-LAYER THEORY

The damping of a mechanical system is given in dimensionless form by the loss coefficient, η_s , which is the ratio of the energy dissipated to the energy stored in the system. That is:

$$\eta_S = \frac{(\Delta W)_S}{2\pi\,(W)_S}$$

where $(\Delta W)_S$ is the energy dissipated per cycle and $(W)_S$ is the energy stored. For single degree of freedom systems, η_s is simply related to the common measures of damping, such as logarithmic decrement δ , and damping ratio ζ . [7]

$$\eta_S = 2\zeta = \frac{\delta}{\pi} .$$

In this study, we apply constrained damping layers to both surfaces of the basic structure. This will give us a symmetric configuration which is simpler to analyse and the general result will be the same as for a single surface treatment. We assume that all of the damping in the constrained viscoelastic layer is attributable to shear strain and the resultant energy dissipation. The ability of the constraining layer to induce shear strain in the constrained layer without itself experiencing excessive stretching is one of the important characteristics of the damping configuration.

In order to study the interaction between the axial strain in the constraining layer and the shear strain in the constrained layer, we consider the case of the constraining layer cut at regular intervals. (Fig. 1)

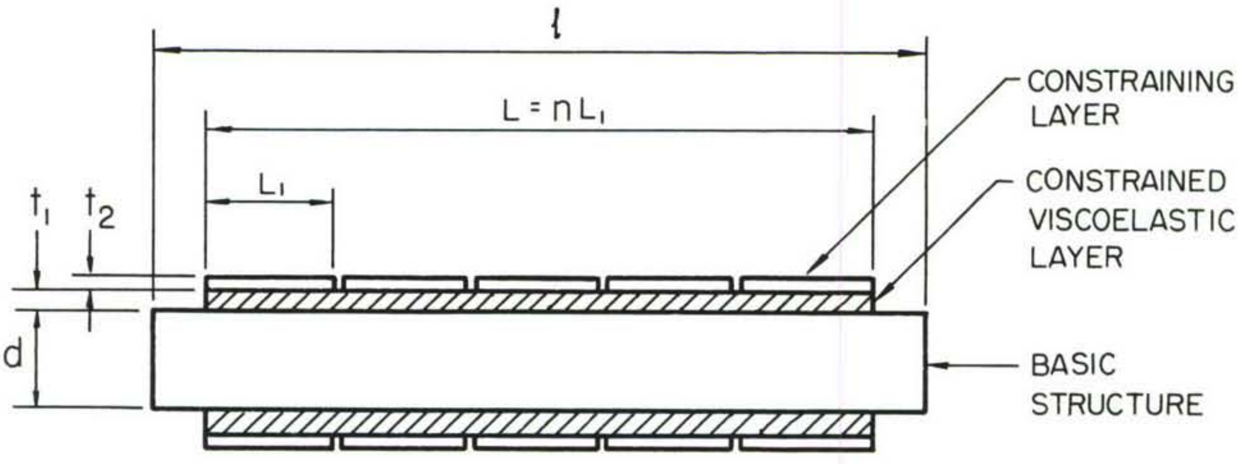


FIG. (1) COMPOSITE STRUCTURE WITH SURFACE TREATMENT.

The following assumptions are made in the analysis which follows:

1. The thicknesses of the constraining layer and of the constrained layer are very small compared to that of the basic structure, thus the bending effects in these layers are neglegible, so that the constraining layer is subjected to tension only and the constrained layer is subjected to shear only.

2. We assume linear behavior of the viscoelastic material; complex notation can be used for its shear modulus

   $$G_1^* = G_1' + iG_1'' = G_1'(1+i\,\eta_G) = G_1(\cos\theta + i\sin\theta)$$

   where the asterisk indicates a complex quantity and

   G_1' is the elastic or storage modulus

   G_1'' is the loss modulus

   $\theta = \tan^{-1}\eta_G$

   η_G is the loss tangent of the viscoelastic material.

3. The constraining material is elastic and dissipates no energy. Its Young's modulus is purely real.

   $E_2'' = 0$ and $E_2 = E_2'$.

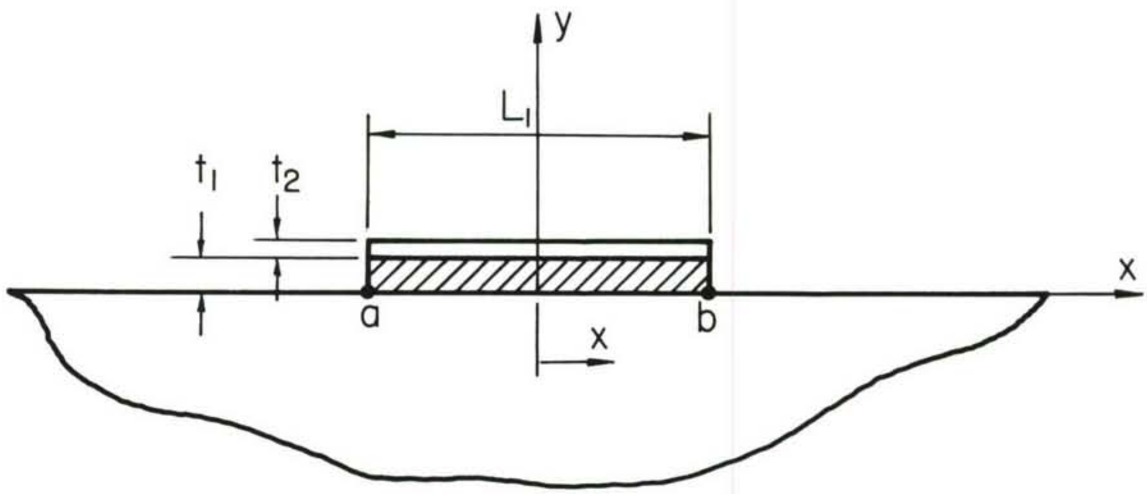
4. Poisson ratio effects are negligible and the one-dimensional problem only is considered.

5. The axial strain is uniform at the interface of the basic structure and the viscoelastic layer.

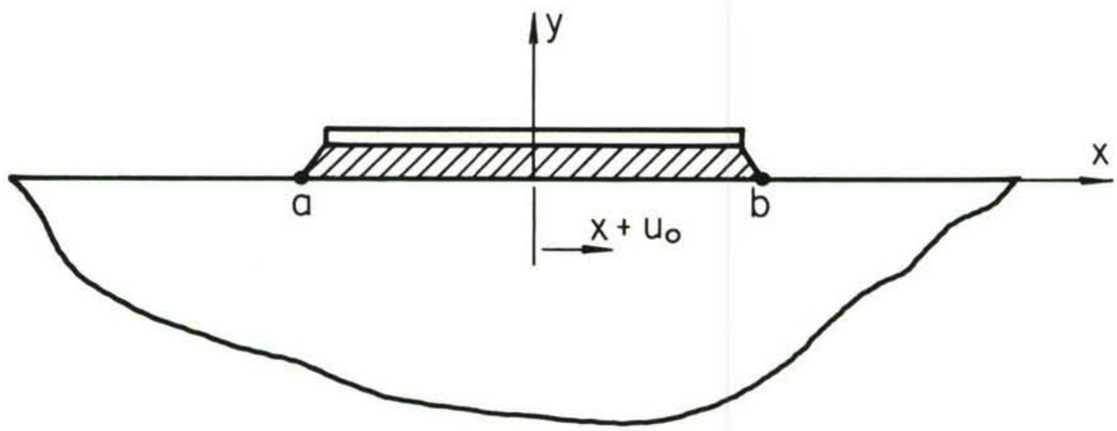
6. Uniform shear strain is assumed through the thickness of the viscoelastic layer.

7. Uniform normal stress is assumed through the thickness of the constraining layer.

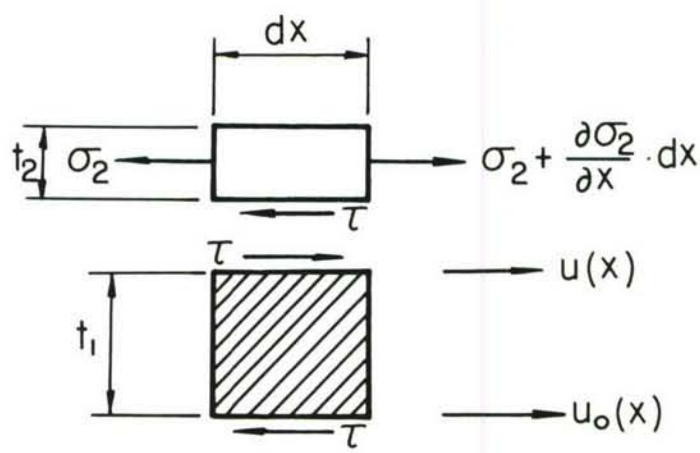
8. The elastic moduli of the viscoelastic layer are small in comparison with those of the constraining layer.



(a) BEFORE DEFORMATION



(b) AFTER DEFORMATION



(c) FREE - BODY DIAGRAM OF AN ELEMENT

FIG. (2) TYPICAL ELEMENT OF CONSTRAINED VISCOELASTIC LAYER APPLIED TO A BASIC STRUCTURE.

From the equilibrium of an element of the constraining layer (Fig. 2-c), we have

$$\int_0^{t_2} \frac{\partial \sigma_2}{\partial x} \cdot dy \cdot dx = \tau \cdot dx$$

or

$$\frac{\partial}{\partial x} \int_0^{t_2} \sigma_2 \cdot dy = \tau \tag{1}$$

$\int_0^{t_2} \sigma_2 \cdot dy$ is the total force acting on the cross section of the constraining layer. If we let $\overline{\sigma}_2$ be the average normal stress in the constraining layer, then

$$\int_0^{t_2} \sigma_2 \cdot dy = \overline{\sigma}_2 \cdot t_2 \quad . \tag{2}$$

Substituting Equation (2) into Equation (1), the equilibrium equation becomes:

$$\frac{\partial \overline{\sigma}_2}{\partial x} \cdot t_2 = \tau \quad . \tag{3}$$

The stress-strain relation in the constraining layer is:

$$\overline{\sigma}_2 = E_2 \frac{\partial u}{\partial x} \tag{4}$$

since we have assumed a one-dimensional problem.

The shear stress-shear strain relation for the viscoelastic layer is:

$$\tau^* = G_1^* \gamma_1^* \quad .$$

Since the shear strain in the constrained layer is constant in y:

$$\gamma_1^* = \frac{u^* - u_o}{t_1}$$

$$\tau^* = G_1^* \cdot \frac{u^* - u_o}{t_1} \tag{5}$$

From assumption (5),

$$u_o = \epsilon_o x$$

where ϵ_0 is the uniform strain in the basic structure varying sinusoidally in time due to flexural vibration.

Substituting Equations (4) and (5) into Equation (3) we obtain the differential equation

$$B_o^{*2} \frac{\partial^2 u^*}{\partial X^2} - u^* = -\epsilon_o X \tag{6}$$

where $B_o^* = \left(t_1 t_2 \frac{E_2}{G_1^*} \right)^{1/2}$ is a system characteristic which has the dimension of length.

It is convenient to use a local coordinate system for which the origin of the abscissa is at the center of one element of the constraining layer; the origin of the ordinate is immaterial for this analysis, and it may be taken at the interface of the basic structure and the viscoelastic layer. (Fig. 2-a).

The boundary conditions for one element of the constraining layer are

$$\frac{\partial u^*}{\partial X} = 0 \qquad \text{AT} \qquad X = \pm \frac{L_1}{2} \tag{7}$$

since there is no normal stress at the ends.

The general solution of Equation (6) is:

$$u^*(x) = \epsilon_o X + A_1 \sinh \frac{X}{B_o^*} + A_2 \cosh \frac{X}{B_o^*} .$$

A_1 and A_2 are determined by the boundary conditions at $X = \pm \frac{L_1}{2}$ (Equation (7))

$$A_1 = - \frac{\epsilon_o B_o^*}{\cosh \frac{L_1}{2 B_o^*}} \quad ; \quad A_2 = 0$$

thus

$$u^*(x) = \epsilon_o \left[X - B_o^* \frac{\sinh \frac{X}{B_o^*}}{\cosh \frac{L_1}{2 B_o^*}} \right] . \tag{8}$$

The energy dissipated per cycle per unit volume of a material in uniform shear is the area within the shear stress-shear strain hysteretic loop. Since we have sinusoidal motion, the time variation of the shear strain may be written:

$$\gamma(t) = R_E (\gamma^* e^{i\omega t})$$

where γ^* is complex. Then

$$\tau(t) = R_E(G_1^* \gamma e^{i\omega t}) .$$

The energy dissipated is

$$R_E\left(\oint \tau d\gamma\right) = R_E\left(\oint G_1^* \gamma d\gamma\right) = \pi G_1'' \gamma^2$$

where $G_1^* = G_1' + i G_1''$.

Since we have assumed that the shear strain is uniform through the thickness of the viscoelastic layer, the energy dissipated per cycle per unit length and width is:

$$d(\Delta W) = \pi t_1 G_1'' \gamma^2$$

Since
$$\gamma^* = \frac{u^* - u_o}{t_1}$$

then, from Equation (8)

$$\gamma^* = \frac{\epsilon_o B_o^*}{t_1} \frac{\sinh \frac{X}{B_o^*}}{\cosh \frac{L_1}{2B_o^*}}$$

and

$$d(\Delta W) = \pi t_1 G_1'' \frac{\epsilon_o^2}{t_1^2} B_o^2 \cdot \left| \frac{\sinh \frac{X}{B_o^*}}{\cosh \frac{L_1}{2B_o^*}} \right|^2 . \tag{9}$$

Writing G_1'' in the form, $G_1'' = G_1 \sin\theta$ in the definition of $\mathbf{B}_o^*$ and using the trigonometric identities for the functions sinh and cosh of a complex argument

$$d(\Delta W) = \frac{2\pi \sin\theta \cdot \epsilon_o^2 t_2 E_2}{\cos\left(\frac{L_1}{B_o}\sin\frac{\theta}{2}\right) + \cosh\left(\frac{L_1}{B_o}\cos\frac{\theta}{2}\right)} \cdot \left[\sin^2\left(\frac{X}{B_o}\sin\frac{\theta}{2}\right) + \sinh^2\left(\frac{X}{B_o}\cos\frac{\theta}{2}\right)\right].$$

This expression can be integrated in explicit form over the length of one element of the constraining layer:

$$\Delta W = \int_{-\frac{L_1}{2}}^{\frac{L_1}{2}} d(\Delta W)\cdot dx$$

$$= \frac{\pi \sin\theta\, \epsilon_o^2\, t_2 E_2}{\cos\left(\frac{L_1}{B_o}\sin\frac{\theta}{2}\right) + \cosh\left(\frac{L_1}{B_o}\cos\frac{\theta}{2}\right)}\left[\frac{B_o}{\cos\frac{\theta}{2}}\sinh\left(\frac{L_1}{B_o}\cos\frac{\theta}{2}\right) - \frac{B_o}{\sin\frac{\theta}{2}}\sin\left(\frac{L_1}{B_o}\sin\frac{\theta}{2}\right)\right] \tag{10}$$

Letting the dimensionless ratio:

$$\omega = \frac{L_1}{B_o} \quad ,$$

ΔW becomes:

$$\Delta W = 2\pi\, \epsilon_o^2\, t_2 E_2 L_1 \cdot \frac{1}{\omega}\left[\frac{\sinh\left(\omega\cdot\cos\frac{\theta}{2}\right)\cdot\sin\frac{\theta}{2} - \sin\left(\omega\cdot\sin\frac{\theta}{2}\right)\cdot\cos\frac{\theta}{2}}{\cosh\left(\omega\cdot\cos\frac{\theta}{2}\right) + \cos\left(\omega\cdot\sin\frac{\theta}{2}\right)}\right]. \tag{11}$$

This can be made dimensionless by dividing by a nominal energy appropriate to the system:

$$W_{NOM.} = \frac{1}{2}\epsilon_o^2\, E_2\, t_2\, L_1 \tag{12}$$

This would be the energy stored in the constraining layer if the whole layer were strained by amount ϵ_o. With this definition we have a dimensionless loss coefficient

$$\eta_1 = \frac{\Delta W}{W_{NOM.}} = 4\pi\cdot\frac{1}{\omega}\left[\frac{\sinh(A)\cdot\sin\frac{\theta}{2} - \sin(B)\cdot\cos\frac{\theta}{2}}{\cosh(A) + \cos(B)}\right] \tag{13}$$

where $A = \omega\cdot\cos\frac{\theta}{2}$ and $B = \omega\cdot\sin\frac{\theta}{2}$ and θ is the loss angle of the viscoelastic material:

$$\tan\theta = \eta_G \; .$$

In Equation (13) η_1 is a function of $\omega\tau$ and η_G only. η_1 is plotted as a function of $\omega\tau$ in Fig. (15)* with η_G as a parameter. Figure (15) shows that for maximum damping

$$\omega\tau = \frac{L_1}{B_o} = 3.28$$

which indicates that for a given viscoelastic material and constraining layer, the length of each element of the constraining layer, L_1, is 3.28 times the characteristic length of the system for optimum damping. Using Equations (12) and (13), we can write ΔW in the form

$$\Delta W = \eta_1 \cdot \frac{1}{2} \epsilon_o^2 E_2 t_2 L_1 \; . \tag{14}$$

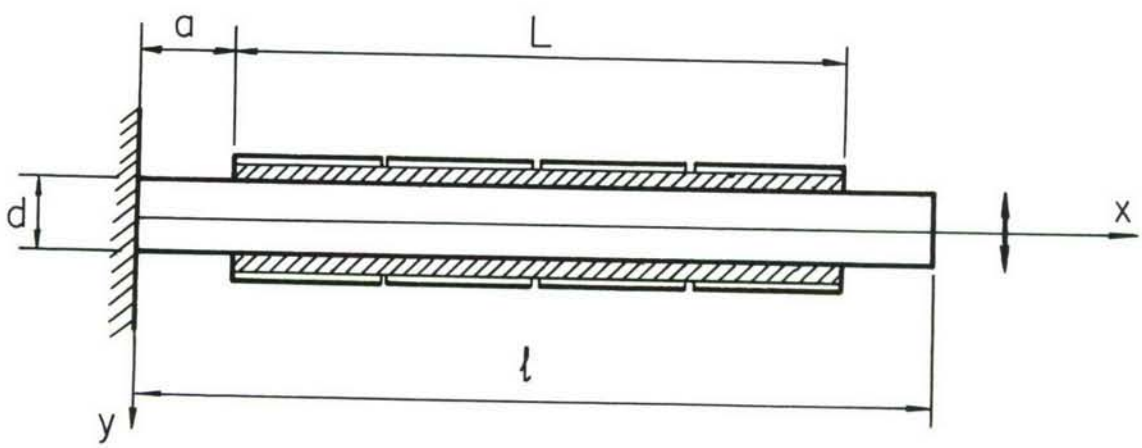


FIG. (3) CANTILEVER BEAM WITH SURFACE TREATMENT.

In order to compare with experimental results, we consider the case of a cantilever beam subjected to sinusoidal flexural vibration with small deformation, Fig. (3); the strain at the interface is

$$\epsilon_o = -\frac{d}{2} \cdot \frac{d^2 y}{dx^2} \tag{15}$$

where d is the thickness of the beam and d^2y/dx^2 is the curvature of the beam. Substituting Equations (14) and (15) into

$$(\Delta W)_L = 2 \int_a^{a+L} \frac{(\Delta W) \cdot dx}{L_1} \quad ,$$

*Figures 15 through 21 are graphs which appear on pages 39 through 45.

we have the energy dissipation per cycle in the constrained visco-elastic layer to be:

$$(\Delta W)_L = \eta_1 \frac{d^2}{4} E_2 t_2 \int_a^{a+L} \left(\frac{d^2y}{dx^2}\right)^2 dx \quad . \tag{16}$$

The factor 2 appears before the integral sign because there is surface treatment on both faces of the basic structure.

The maximum energy stored in the system is

$$\begin{aligned}(W)_s &= \frac{E_b I}{2} \int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx \\ &= \frac{E_b d^3}{24} \int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx\end{aligned} \tag{17}$$

where E_b is the Young's modulus of the basic material. From Equations (16) and (17) we obtain the modified loss coefficient of the system:

$$\eta_L = \frac{(\Delta W)_L}{2\pi (W)_s} = \eta_1 \frac{3 E_2 t_2}{\pi E_b d} \cdot \frac{\int_a^{a+L} \left(\frac{d^2y}{dx^2}\right)^2 dx}{\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx} \quad . \tag{18}$$

The vibratory shape of a uniform cantilever is [8]

$$y(x) = \cosh \lambda x - \cos \lambda x - \bar{\gamma} (\sinh \lambda x - \sin \lambda x)$$

and its curvature is

$$\frac{d^2y}{dx^2} = \lambda^2 \left[\cosh \lambda x + \cos \lambda x - \bar{\chi} (\sinh \lambda x + \sin \lambda x)\right] \tag{19}$$

where $\lambda l = 1.875$ and $\bar{\chi} = \dfrac{\sinh \lambda l - \sin \lambda l}{\cosh \lambda l + \cos \lambda l}$ for the fundamental mode of vibration. In order to compare with the experimental results, we can substitute Equation (19) into Equation (18) and evaluate the integrals explicitly. The modified loss coefficient of the system, η_L, can then be found in terms of η_1

(Equation (13)) and the geometry and material properties of the beam and the constraining layer.

Equation (18) shows that the loss coefficient of the system depends on the stiffness of the constraining layer and the material property of the basic structure. It does not depend explicitly on the shear modulus of the constrained viscoelastic material. The above integral is evaluated for an explicit case in Section IV and the results are compared with measured values.

III

Multiple - Layer Theory

The amount of damping in structures studied in the previous section can be increased by applying more than one constrained viscoelastic layer to the surface of the basic structure as shown in Fig. (4).

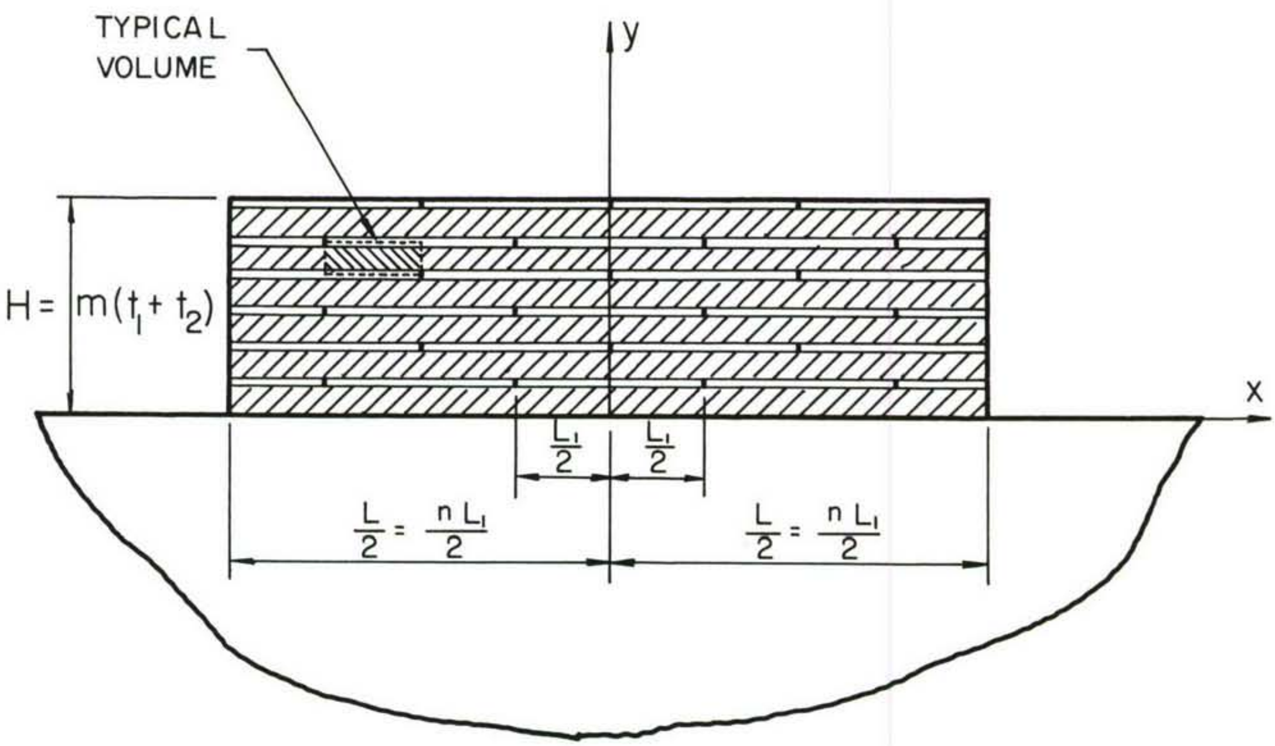


FIG. (4) MULTIPLE LAYER SURFACE TREATMENT.

If there is more than one constrained viscoelastic layer, there are viscoelastic layers on both sides of the constraining layer and interactions are induced between them. The governing equations of equilibrium can still be written for each individual layer, but to solve this set of equations for a large number of layers would be tedious. For convenience, we replace a repetitive element which is typical of the multiple-layered treatment by an equivalent homogeneous orthotropic material with the same force-deformation relationship. The equilibrium condition for this typical volume is shown in Fig. (5).

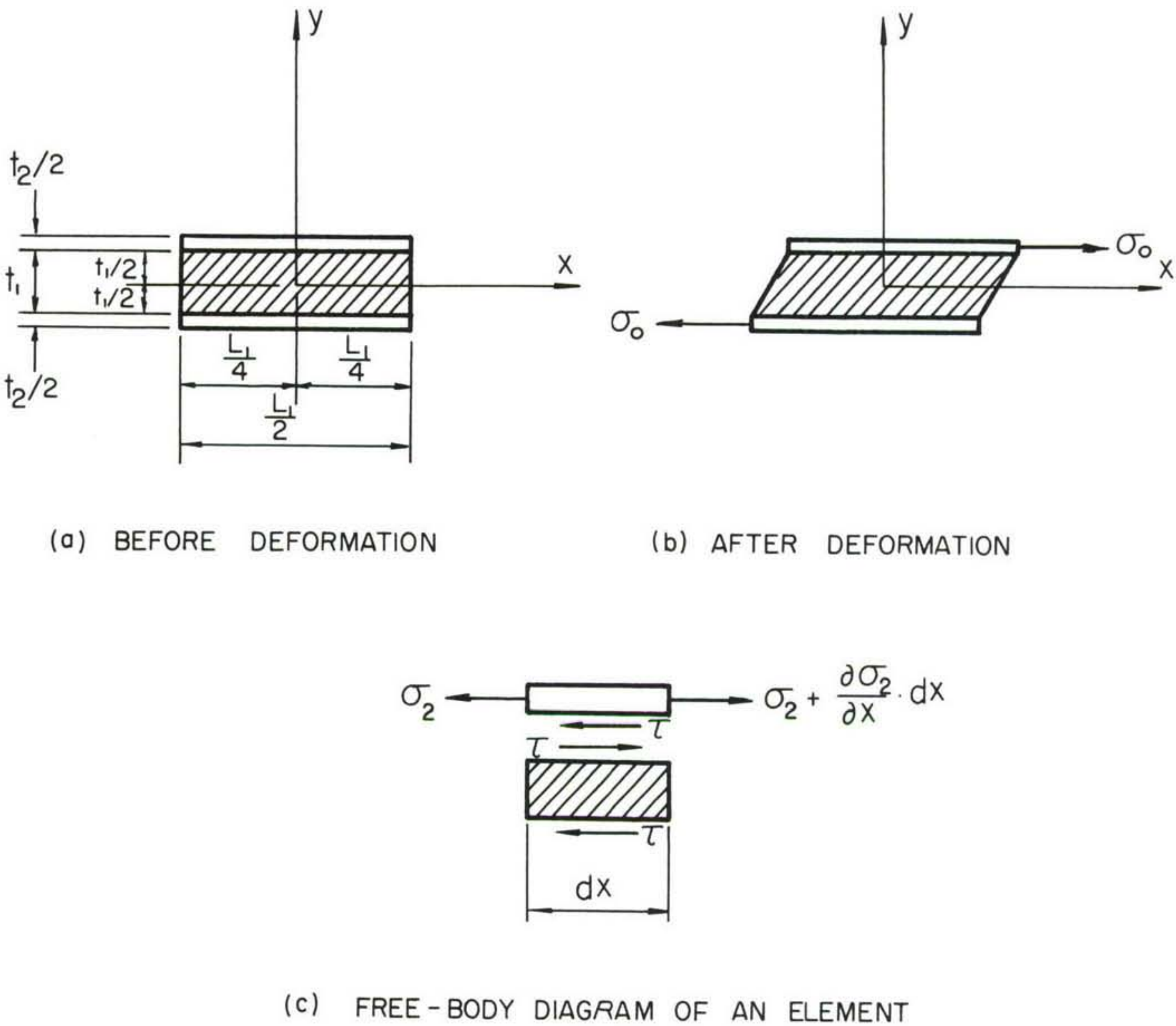


FIG. (5) EQUILIBRIUM OF A TYPICAL VOLUME.

We follow the same assumptions made for single layer analysis in Section II. From the equilibrium of an element of the visco-elastic layer we have the differential equation of equilibrium in the x-direction:

$$\frac{\partial \sigma_1}{\partial x} + \frac{\partial \tau}{\partial y} = 0 .$$

Since the strain for the constraining layer, ϵ_2 , is of the same order as the strain for the viscoelastic layer, ϵ_1 , and $E_2 >> E_1$ (assumption (8)), then $\sigma_2 >> \sigma_1$, and σ_1 is negligible (assumption (1)). Then

$$\frac{\partial \tau}{\partial y} = 0 \qquad \text{or} \qquad \frac{\partial \gamma}{\partial y} = 0 \tag{20}$$

From assumption (4) the deformation in the y-direction is negligible,

$$\gamma = \frac{\partial u}{\partial y}$$

and Equation (20) becomes

$$\frac{\partial \gamma}{\partial y} = \frac{\partial^2 u}{\partial y^2} = 0 \ .$$

With these assumptions, the deformation, u , in the x-direction must be linear in y.

From Fig. (5-b) the axial deformation is anti-symmetric, i.e.,

$$u(x, y) = -u(-x, -y) \ ,$$

we can write u(x,y) in the form:

$$u(x,y) = y \cdot f_1(x) + f_2(x) \tag{21}$$

If $f_1(x)$ is symmetric in x and $f_2(x)$ is anti-symmetric in x, Equation (21) is the most general anti-symmetric function which is linear in y. From Equation (21),

$$\gamma(x) = f_1(x) \ . \tag{22}$$

Substituting Equations (21) and (22) into the equation of equilibrium of the constraining layer

$$\frac{\partial \sigma_2}{\partial x} \cdot \frac{t_2}{2} = \tau$$

and using the stress-strain relations for the constraining layer and the viscoelastic layer

$$\sigma_2 = E_2 \frac{\partial u}{\partial x}\left(x, \frac{t_1}{2}\right)$$

and $$\tau^* = G_1^* \gamma^* ,$$

we get

$$t_1 t_2 \frac{E_2}{4G_1^*} f_1''(x) - f_1(x) = - \frac{E_2 t_2}{2 G_1^*} f_2''(x) . \tag{23}$$

The left hand side is symmetric by definition, therefore the right hand side must be symmetric. But $f_2''(x)$ is anti-symmetric unless it is zero. Equation (23) is therefore:

$$\frac{1}{4} L_o^{*2} f_1''(x) - f_1(x) = 0 \tag{24}$$

where $L_o^* = \left(t_1 t_2 \frac{E_2}{G_1^*} \right)^{\frac{1}{2}}$ is a system characteristic which has the dimension of length. The symmetric solution to Equation (24) is

$$f_1(x) = A_1 \cosh \frac{2X}{L_o^*} . \tag{25}$$

Since $f_2''(x) = 0$ and $f_2(x)$ is anti-symmetric,

$$f_2(x) = A_2 X . \tag{26}$$

Substituting Equations (25) and (26) into Equation (21) we get:

$$u^*(x, \frac{t_1}{2}) = A_1 \frac{t_1}{2} \cosh \frac{2X}{L_o^*} + A_2 X . \tag{27}$$

The constants A_1 and A_2 are evaluated from the boundary conditions:

(i) Stress at $\left(\frac{L_1}{4}, \frac{t_1}{2} \right)$ is σ_o

$$\frac{\partial u^*}{\partial x} \left(\frac{L_1}{4}, \frac{t_1}{2} \right) = \frac{\sigma_o}{E_2}$$

(ii) The end $(-\frac{L_1}{4}, \frac{t_1}{2})$ is stress free

$$\frac{\partial u^*}{\partial x}(-\frac{L_1}{4}, \frac{t_1}{2}) = 0 \ .$$

We obtain

$$A_1 = \frac{\sigma_o L_o^*}{2E_2 t_1 \,\mathrm{SINH}\,\frac{L_1}{2L_o^*}} \quad ; \quad A_2 = \frac{\sigma_o}{2E_2}$$

so that Equation (27) becomes

$$u^*(x, \frac{t_1}{2}) = \frac{\sigma_o}{2E_2}\left[x + \frac{L_o^*}{2\,\mathrm{SINH}\frac{L_1}{2L_o^*}} \cdot \mathrm{COSH}\,\frac{2x}{L_o^*} \right] . \tag{28}$$

This is the deformation in the constraining layer in the x-direction.

We now define an equivalent homogeneous medium with the same average deformation as the composite non-homogeneous material. For thin layers, σ_y is negligible; the effective moduli for σ_x and τ_{xy} are:

$$\sigma_E^* = E_E^* \cdot \epsilon_E^*$$

and

$$\tau_E^* = G_E^* \cdot \gamma_E^*$$

ϵ_E^* is the total displacement over the quarter length divided by $L_1/4$ (Fig. (5-b)) and γ_E^* is the average displacement in one thickness of the viscoelastic layer divided by the thickness (t_1+t_2).

The effective stress is

$$\sigma_E^* = \frac{\sigma_o \ t_2}{2(t_1 + t_2)}$$

and the effective strain is

$$\epsilon_E^* = \frac{u^*(\frac{L_1}{4}, \frac{t_1}{2})}{\frac{L_1}{4}}$$

Evaluating Equation (28) at $x = \frac{L_1}{4}$,

$$\epsilon_E^* = \frac{\sigma_o}{2E_2}\left[1 + \frac{1}{\alpha^*} \cdot \mathrm{COTH}\ \alpha^* \right]$$

where $\alpha^* = \frac{L_1}{2L_o^*}$.

Therefore,

$$E_E^* = \frac{\sigma_E^*}{\epsilon_E^*}$$

$$= E_2 \left(\frac{t_2}{t_1 + t_2}\right) \cdot \left[1 + \frac{1}{\alpha^*} \cdot \mathrm{COTH}\ \alpha^* \right]^{-1} \tag{29}$$

is the effective Young's modulus of the equivalent homogeneous material.

Since the equivalent homogeneous material has the same force-deformation relationship as for the typical volume of the multiple layer treatment, the effective shear stress is:

$$\tau_E^* = G_1^* \cdot \frac{u^*(x, \frac{t_1}{2})}{t_1}$$

and since

$$\gamma_E^* = \frac{u^*(x, \frac{t_1}{2})}{t_1 + t_2}$$

then

$$G_E^* = \frac{\tau_E^*}{\gamma_E^*}$$

$$= G_1^* \cdot \frac{t_1 + t_2}{t_1} \tag{30}$$

This is the effective shear modulus of the equivalent homogeneous material. Equations (29) and (30) show that the effective moduli of the equivalent homogeneous medium are determined by the dimensions and material properties of the constituent layers of the non-homogeneous material.

The free-body diagram of an element of the equivalent homogeneous material is shown in Fig. (6).

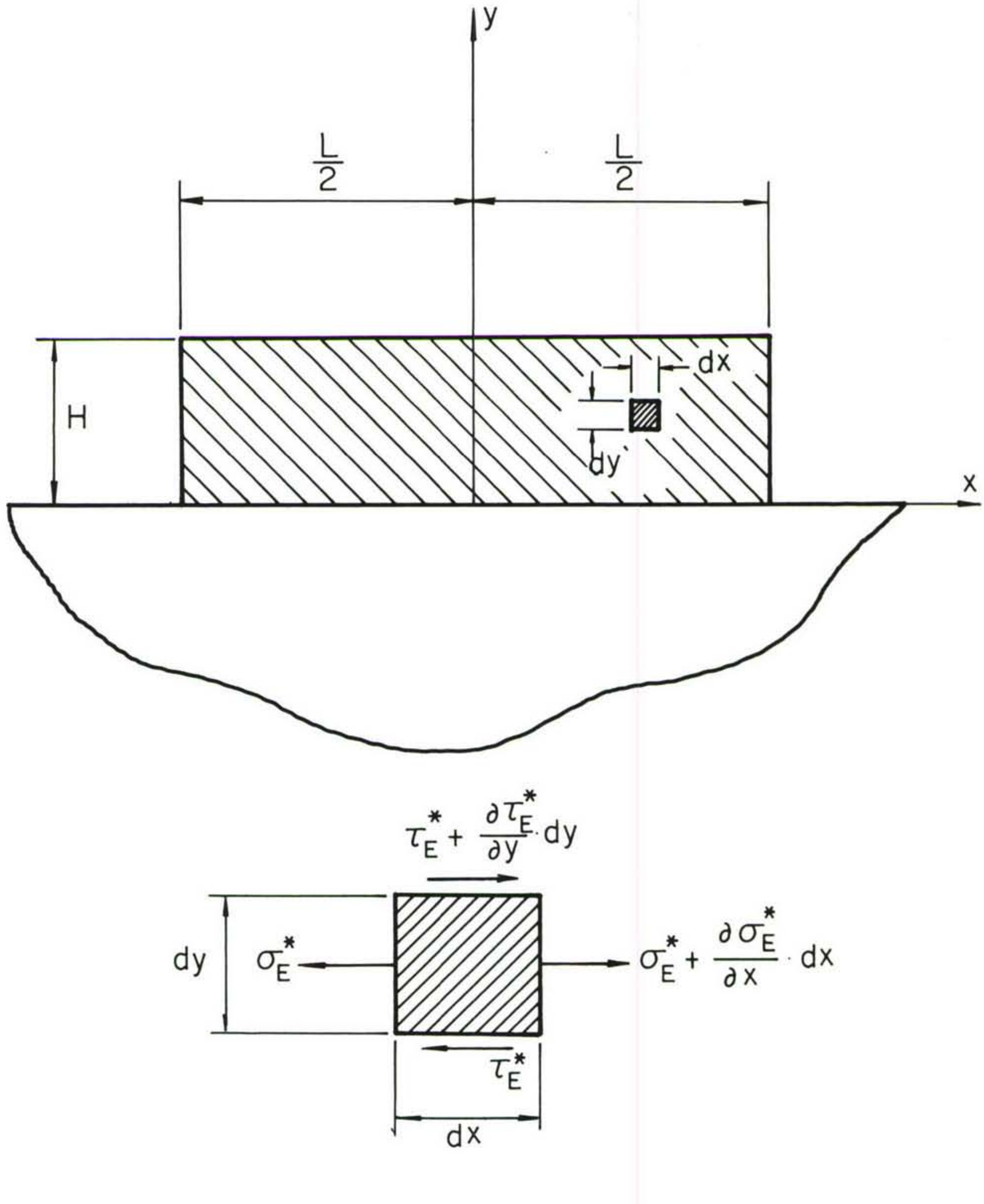


FIG. (6) EQUILIBRIUM OF AN ELEMENT OF THE EQUIVALENT HOMOGENEOUS MEDIUM.

The equation of equilibrium for the element is:

$$\frac{\partial \sigma_E^*}{\partial x} + \frac{\partial \tau_E^*}{\partial y} = 0 \tag{31}$$

The stress-strain relations are:

$$\sigma_E^* = E_E^* \cdot \frac{\partial u^*}{\partial x}$$

and

$$\tau_E^* = G_E^* \cdot \frac{\partial u^*}{\partial y} \quad ,$$

and Equation (31) can be written as

$$\frac{\partial^2 u^*}{\partial x^2} + C^{*2} \frac{\partial^2 u^*}{\partial y^2} = 0 \tag{32}$$

where $C^{*2} = \frac{G_E^*}{E_E^*}$.

Using the coordinate system as shown in Fig. (6), the boundary conditions are:

(i) The strain at the interface is uniform:

$$u^*(x, 0) = \epsilon_o x \quad ,$$

(ii) The shear stress on the top surface is zero:

$$\frac{\partial u^*}{\partial y}(x, H) = 0 \quad ,$$

(iii) Normal stresses at the ends vanish:

$$\frac{\partial u^*}{\partial x}\left(\pm \frac{L}{2}, y\right) = 0 \; .$$

The general solution to Equation (32) satisfying boundary conditions (i) and (ii) is:

$$u^*(x, y) = \epsilon_o x + \sum_{k\ \mathrm{ODD}} A_k \sinh\left(\frac{k C^*}{B} x\right) \cdot \sin\left(\frac{k\pi}{2H} y\right) . \tag{33}$$

Substituting Equation (33) into Equation (32) we find

$$B = \frac{2H}{\pi} \quad ,$$

and A_k is determined by the boundary condition (iii)

$$\epsilon_o + C^* \sum_{k\ \mathrm{ODD}} A_k \frac{k\pi}{2H} \cdot \cosh\left(\frac{k\pi C^* L}{4H}\right) \cdot \sin\left(\frac{k\pi}{2H} y\right) = 0$$

or

$$\frac{\epsilon_o}{C^*} = \sum_{k\ \mathrm{ODD}} \left[-A_k \cdot \frac{k\pi}{2H} \cdot \cosh\left(\frac{k\pi C^* L}{4H}\right)\right] \cdot \sin\left(\frac{k\pi}{2H} y\right)$$

where

$$-A_k \cdot \frac{k\pi}{2H} \cdot \cosh\left(\frac{k\pi C^* L}{4H}\right) = \frac{2\,\epsilon_o}{C^* H} \int_0^H \sin\left(\frac{k\pi}{2H} y\right) \cdot dy$$

is the Fourier sine coefficient of the function $\frac{\epsilon_o}{C^*}$.
Since

$$\int_0^H \sin\left(\frac{k\pi}{2H} y\right) \cdot dy = \frac{2H}{k\pi} \quad ,$$

$$A_k = -\frac{8H\epsilon_o}{C^* k^2 \pi^2 \cdot \cosh\left(\frac{k\pi C^* L}{4H}\right)} \quad .$$

The displacement solution (33) becomes then

$$u^*(x,y) = \epsilon_o \left[x - \sum_{k\ \mathrm{ODD}} \frac{8H \sinh\left(\frac{k\pi C^*}{2H} x\right)}{C^* k^2 \pi^2 \cosh\left(\frac{k\pi C^* L}{4H}\right)} \cdot \sin\left(\frac{k\pi}{2H} \cdot y\right)\right]. \tag{34}$$

The strain energy in the surface treatment is the work done on it through the interface between the basic structure and the treatment. Since the only force acting on the treatment is the shear stress at the interface, the strain energy is the work done by this shear force:

$$\begin{aligned} W^* &= \int_{-\frac{L}{2}}^{\frac{L}{2}} \tau_E^*(x,0) \cdot u^*(x,0) \cdot dx \\ &= 2\int_0^{\frac{L}{2}} \tau_E^*(x,0) \cdot u^*(x,0) \cdot dx \, , \end{aligned} \tag{35}$$

since the surface treatment is symmetric about $x = 0$ (Fig.(6)). Substituting the stress-strain relation

$$\tau_E^* = G_E^* \gamma_E^* \quad ,$$

where

$$\gamma_E^* = \frac{\partial u^*}{\partial y}$$

$$= \sum_{k\,ODD} \frac{4\,\epsilon_o \,\mathrm{SINH}(\frac{k\pi C^*}{2H} x)}{C^* k\pi \cdot \mathrm{COSH}(\frac{k\pi C^* L}{4H})} \cdot \cos(\frac{k\pi}{2H} y)$$

and $u^*(x,0) = \epsilon_o x$, for uniform strain at the interface, into Equation (35) and integrating along the length, we have

$$W^* = 8\epsilon_o^2 H \cdot L \cdot E_E^* \sum_{k\,ODD} \frac{1}{k^2\pi^2} \left[1 - \frac{1}{k\beta^*} \cdot \mathrm{TANH}(k\beta^*)\right] \tag{36}$$

where

$$\beta^* = \frac{L}{H} \cdot \frac{\pi}{4} \cdot C^* = \frac{n}{m} \cdot \frac{L_1}{t_1 + t_2} \cdot \frac{\pi}{4} \cdot C^*$$

and $n = \frac{L}{L_1}$ is the number of individual elements of the constraining layer,

$m = H/(t_1+t_2)$ is the number of layers of the surface treatment.

The energy dissipation per cycle is then

$$\Delta W = \pi \cdot I_M (W^*)$$

or

$$\Delta W = 8\pi\,\epsilon_o^2 H \cdot L \cdot E_2 (\frac{t_2}{t_1 + t_2}) \cdot I_M \left\{ \left[1 + \frac{1}{\alpha^*} \cdot \mathrm{COTH}\,\alpha^*\right]^{-1} \cdot \sum_{k\,ODD} \frac{1}{k^2\pi^2} \left[1 - \frac{1}{k\beta^*} \cdot \mathrm{TANH}(k\beta^*)\right] \right\} \tag{37}$$

This result is the same as would be found by evaluating

$$\Delta W = \int_V \pi \cdot G_E'' \cdot \gamma_E^2 \cdot dV \quad .$$

as was used for single layer analysis.

Equation (37) can be made dimensionless by dividing by a nominal energy appropriate to the system

$$W_{NOM.} = \frac{1}{2}\epsilon_o^2 E_2 \cdot H \cdot L \cdot \left(\frac{t_2}{t_1 + t_2}\right) \tag{38}$$

The dimensionless loss coefficient is then

$$\eta_1 = \frac{\Delta W}{W_{NOM.}}$$

$$= 16\pi \cdot I_M \left\{ \left[1 + \frac{1}{\alpha^*} \text{COTH } \alpha^* \right]^{-1} \cdot \sum_{k \text{ ODD}} \frac{1}{k^2 \pi^2} \left[1 - \frac{1}{k\beta^*} \cdot \text{TANH}(k\beta^*) \right] \right\} \tag{39}$$

Equation (39) indicates that the dimensionless loss coefficient η_1 is a function of α^* and β^*. From the definition of α^*,

$$\alpha^* = \frac{L_1}{2L_o^*} = \frac{L_1}{2L_o}\left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right) = \frac{\xi}{2}\left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right) \tag{40}$$

where $\xi = \frac{L_1}{L_o}$

and $\text{TAN}\,\theta = \eta_G$

Also $\beta^* = \frac{L}{H}\cdot\frac{\pi}{4}\cdot C^* = \frac{n}{m}\cdot\frac{L_1}{t_1 + t_2}\cdot\frac{\pi}{4}\cdot C^*$

where

$$C^* = \left(\frac{G_E^*}{E_E^*}\right)^{\frac{1}{2}}$$

$$= \frac{t_1 + t_2}{L_o^*}\left[1 + \frac{1}{\alpha^*}\cdot \text{COTH } \alpha^*\right]^{\frac{1}{2}},$$

using Equations (29) and (30). Then

$$\beta^* = \frac{n}{m}\cdot\frac{\pi}{2}\cdot\alpha^*\left[1 + \frac{1}{\alpha^*}\cdot\text{COTH } \alpha^*\right]^{\frac{1}{2}} \tag{41}$$

Figures (16) to (19) show η_1 as a function of ξ for different η_G with n/m as a parameter. In the calculations for these plots only five terms were used in the series in Equation (39). The error involved in truncating after five terms of the infinite series depends upon the values of ξ and n/m. This error is a minimum when ξ is optimum and appears to decrease at optimum for n/m larger or smaller than 1. The following table shows the percentage difference between the 5 term and the 15 term approximation; the maximum discrepancy for optimum ξ is about 4% at n/m = 1

n/m \ ξ	0.5	4.0	16.0
0.1	36.0%	7.0%	0.9%
1.0	5.6%	4.8%	15.0%
10.0	3.0%	2.8%	19.0%

For comparison purposes, we can derive the loss coefficient for the case of a cantilever beam in flexural vibration with small deformation in the same manner as in Section II; the modified loss coefficient for the system is:

$$\eta_L = \eta_1 \cdot \frac{3 E_2 m \cdot t_2}{\pi E_b d} \cdot \frac{\int_a^{a+L} \left(\frac{d^2y}{dx^2}\right)^2 dx}{\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx} \tag{42}$$

This is exactly the same as Equation (18) with t_2, for single layer treatment, replaced by mt_2 for multiple layer treatment.

IV

Experimental Results and Comparison with Theoretical Predictions

Vibration decay measurements were made on a number of different cantilever beams each with the same total length of the constrained visco-elastic layer damping treatment but with different element lengths for comparison with the loss coefficient η_L given by Equation (18) for a single layer. For the multiple layer treatment, one through eight layers were used, each with the same number of elements . The test configuration used in this experimental program is shown in Fig. 7).

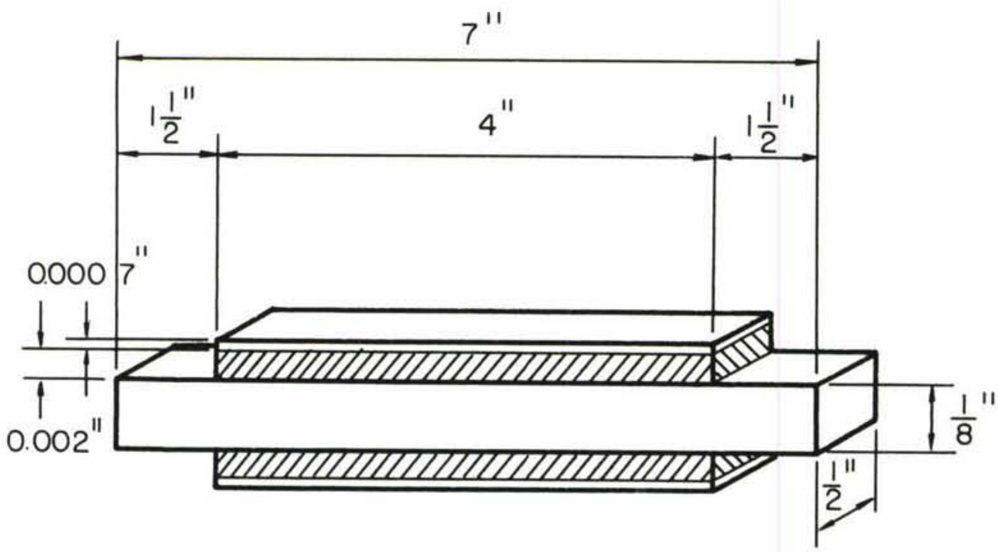


FIG. (7) TEST CONFIGURATION OF SINGLE LAYER SURFACE TREATMENT.

The basic structure was a C1018 steel beam 7 inches long, ½ inch wide, and 1/8 inch thick. 0.0007 inch thick aluminum foil was used as constraining layer. For the constrained viscoelastic layer, we used No. 466, 3M adhesive. Material properties of this adhesive were found from the master curves furnished by 3M Company. $\eta_G = 1.5$, $G_1 = 250$ at a frequency of 66 cps. and at room temperature were used in calculations. The surface treatment was applied to the middle 4 inches of both faces of the basic structure. The test specimen was clamped at one end to a massive base isolated from the floor by foam rubber springs. The free length of the specimen was 6 inches. An accelerometer attached to the free end of the canti-lever beam gave an electric signal proportional to the amplitude of free vibration. A magnetic driver was used to drive the beam at the required amplitude (Fig. 8).

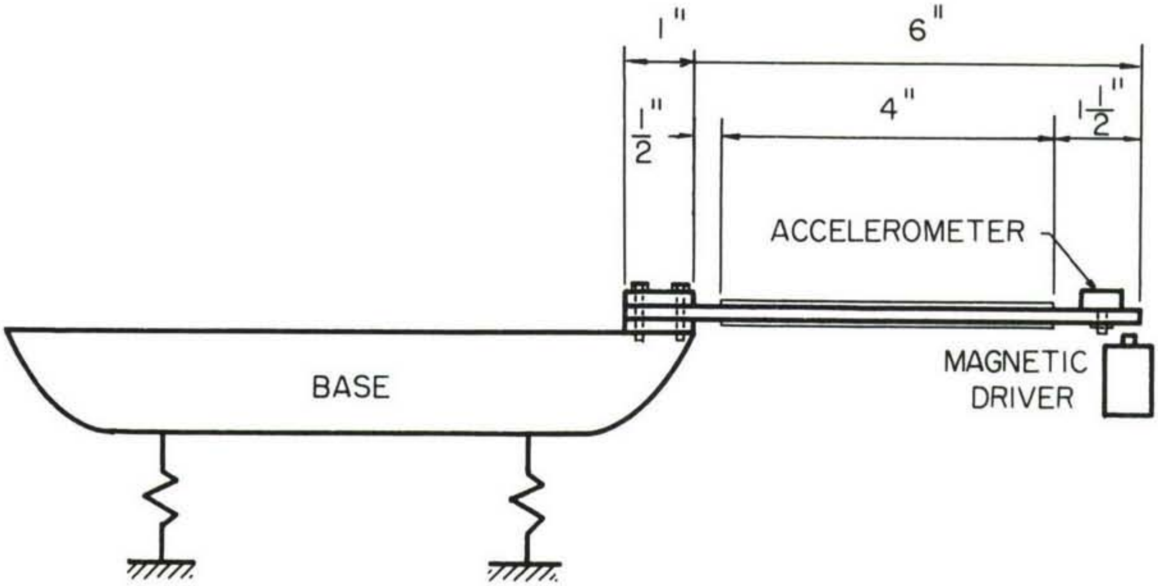


FIG. (8) SPECIMEN MOUNTING.

After steady state resonant vibration of the required amplitude was reached, power to the driver was cut off and the logarithmic decrement was measured. The equipment used for the measurement of decrement is shown in the block diagram (Fig. 9).

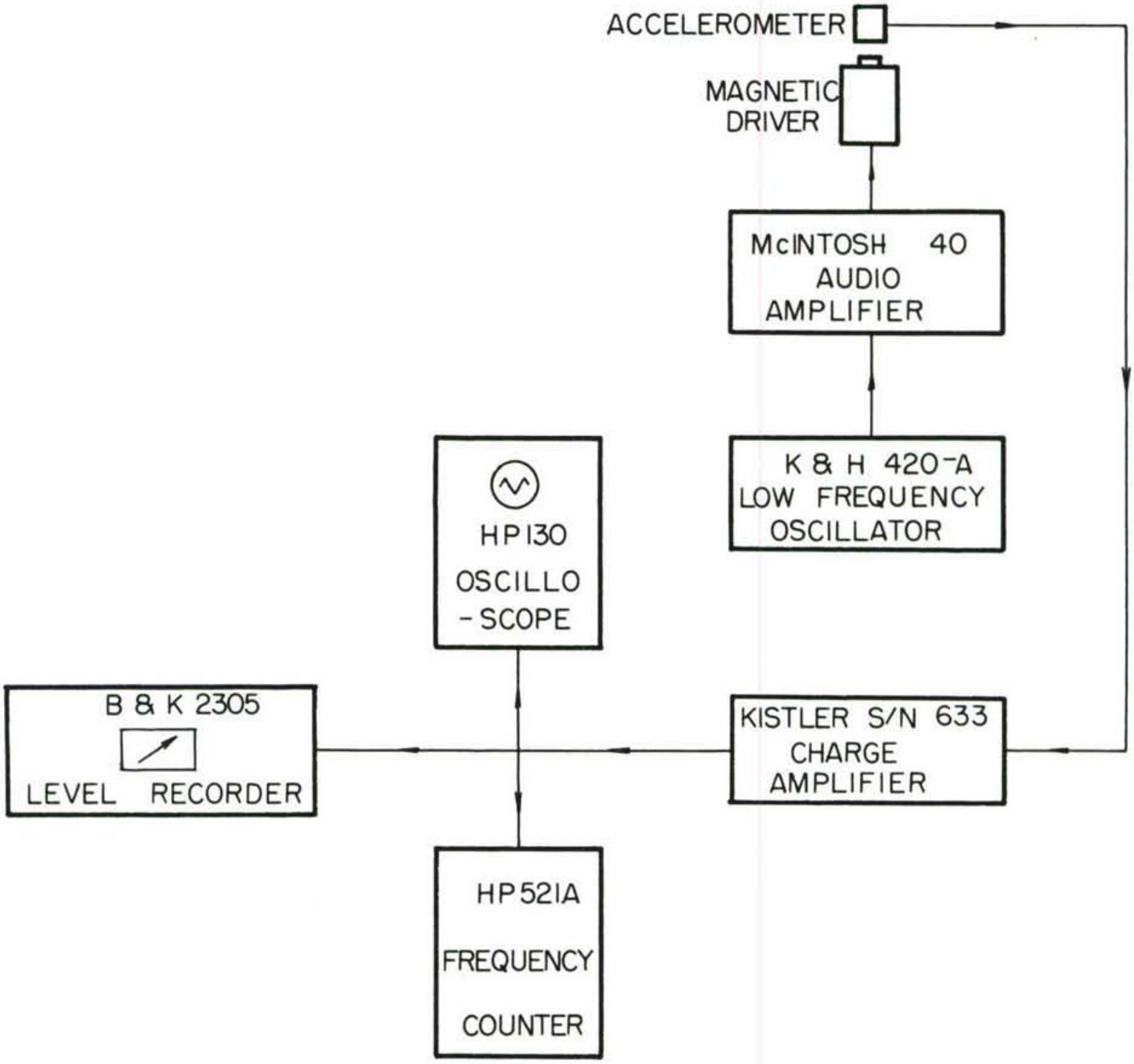


FIG. (9) BLOCK DIAGRAM OF INSTRUMENTATION

(a) The low frequency oscillator and audio amplifier provided power to the magnetic driver to drive the specimen at its fundamental natural frequency.

(b) The accelerometer gives an electrical signal proportional to the amplitude of vibration of the tip.

(c) The charge amplifier amplifies the signal for counter readings, oscilloscope monitoring and recorder input.

(d) The counter reads the natural frequency of the vibrating beam.

(e) The voltage from the preamplifier is displayed on an oscilloscope to indicate the stress level of the vibrating beam and to monitor the wave shape of the signal.

(f) The level recorder gives the envelope of the decay curve in a logarithmic scale. The slope of this curve is directly proportional to the loss coefficient

$$\eta_B \text{ OR } \eta_S = \frac{\log_e 10}{20\pi f} \cdot \frac{d\,dB}{dt} \qquad \text{where } \frac{d(dB)}{dt}$$

is the slope of the decay curve in dB per second. Since the energy dissipation in the constrained viscoelastic layer can not be measured directly, we must find it from the energy dissipation in the bare specimen without surface treatment, $(\Delta W)_B$, and the energy dissipation in the test specimen with surface treatment, $(\Delta W)_S$. The energy dissipation in the constrained viscoelastic layer alone is the difference between these two, $(\Delta W)_L = (\Delta W)_S - (\Delta W)_B$

For very thin layer surface treatment, we assume that the maximum energy stored in the system, is the same for both the bare specimen and the test specimen with surface treatment, i.e., $(W)_S = (W)_B$
Then the loss coefficients for these two cases are:

$$\eta_S = \frac{(\Delta W)_S}{2\pi (W)_S} \quad \text{AND} \quad \eta_B = \frac{(\Delta W)_B}{2\pi (W)_B} = \frac{(\Delta W)_B}{2\pi (W)_S}$$

Since the modified loss coefficient of the viscoelastic material in this system is defined as (18bis)

$$\eta_L = \frac{(\Delta W)_L}{2\pi (W)_S},$$

it is the difference between the loss coefficient of the bare specimen, η_B, and the loss coefficient of the test specimen with surface treatment, η_S:

$$\eta_L = \eta_S - \eta_B .$$

Using the measured values of η_B and η_S, and Equation (18)

$$\eta_1 = \eta_L \cdot \frac{\pi \cdot E_b \cdot d}{3 \cdot E_2 \cdot t_2} \cdot \frac{\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 \cdot dx}{\int_a^{a+L} \left(\frac{d^2y}{dx^2}\right)^2 \cdot dx} \tag{43}$$

together with Equation (19) and the dimensions and material properties of the test specimen used, the equivalent loss coefficient for uniform strain is:

$$\overline{\eta}_1 = 0.803 \times 10^{-3} (\eta_S - \eta_B) \tag{44}$$

The constraining layer was cut at regular intervals and measurements were made for a number of different values of L_1. The values of the dimensionless loss coefficient η_1 found from the measured $(\eta_S - \eta_B)$ are plotted in Fig. (20). For the configuration used, $t_1 = 0.002$ in., $t_2 = 0.0007$ in., G_1 250 psi, $E_2 = 10 \times 10^6$psi; therefore $L_0 = 0.236$ in. A theoretical curve for η_1 as found from Equation (13) using $\eta_G = 1.5$ is shown in the same figure.

For multiple layer treatment more than one constrained visco-elastic layer is applied to each of the surfaces of the basic strucutre. Each subsequent constraining layer overlaps the previous one; in this particular test a length of $L_1 = 2/3$ inch was used for each element of the constraining layer except at the ends. (Fig. 10).

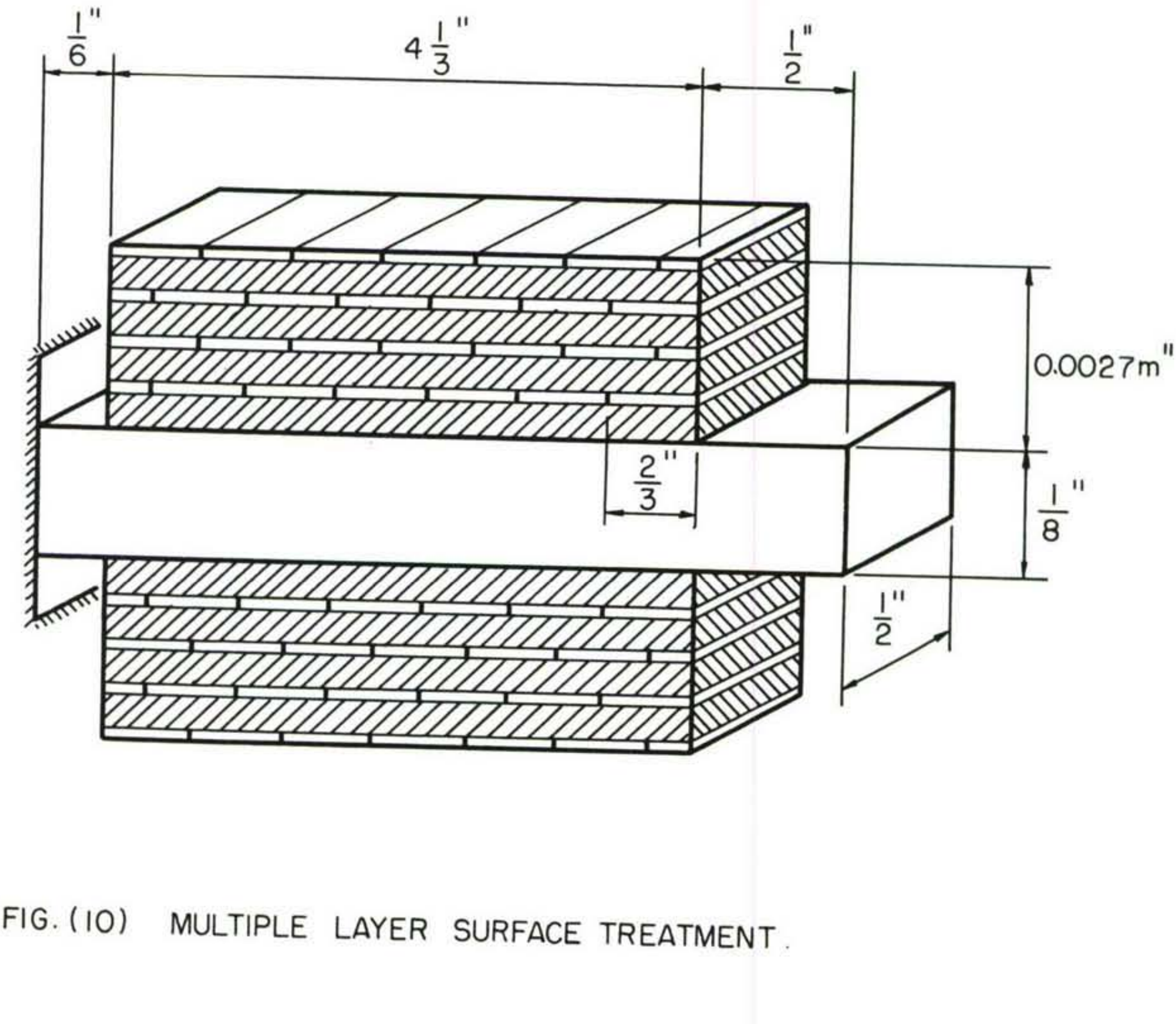


FIG. (10) MULTIPLE LAYER SURFACE TREATMENT.

The measured loss coefficient, ($\eta_s - \eta_B$), is plotted against the number of layers, m, in Fig. (21). From equation (42), using the dimensions and properties of the test specimen (42)

$$(\eta_S - \eta_B) = 1.24 \times 10^{-3} \cdot m \cdot \eta_1 \tag{45}$$

where η_1 is found from Fig. (18). L_o is still 0.236 inch, and L_1 is 0.667 inch, therefore ξ = 2.82. In this case n , and number of elements, is 6.5 and m is the number of constraining layers. For m = 1, η_1 is given by single layer analysis. Fig. (20) ($\eta_s - \eta_B$) is plotted in Fig. (21) for m = 1 to 8 for values calculated from Equation (45). Since the values of η_1 for m ≥ 2 are not found by the same method as that for m = 1, the points do not fall on a smooth curve.

V

DISCUSSION

In the analysis presented above, it has been assumed that the viscoelastic layer has a much smaller elastic modulus than the constraining layer and that the constraining layer is non-dissipative. The geometric effects which change the overall damping primarily influence the distribution of strain and extension in the constraining layer and these interactions may be best understood by examining the appropriate functions in some detail.

For the single layer treatment, the axial stress in the constraining layer is:

$$\sigma_2^* = E_2 \frac{\partial u^*}{\partial x}$$

where u^* is the displacement in the x direction. Using Equation (8), we have

$$\sigma_2^* = E_2 \epsilon_o \left[1 - \frac{\cosh \frac{x}{B_o^*}}{\cosh \frac{L_1}{2B_o^*}} \right] \tag{46}$$

The shear stress in the viscoelastic layer is found from Equations (5) and (8):

$$\tau^* = G_1^* \frac{\epsilon_o B_o^*}{t_1} \cdot \frac{\sinh \frac{x}{B_o^*}}{\cosh \frac{L_1}{2B_o^*}} \tag{47}$$

Figure (11) shows $\sigma_2^*/(E_2\epsilon_o)$ and $\tau^*/(G_1^* \frac{\epsilon_o B_o^*}{t_1})$

as functions of x with L_1 as a parameter. For large L_1, the central portion of the constraining layer undergoes the same axial strain, ϵ_o, as in the basic structure, there is no shear in the viscoelastic layer away from the ends and the damping is small. (Fig. 11a). For very small L_1, the elements of the constraining layer exert no constraint on the underlying viscoelastic layer, there is little shear strain and the damping is again small (Fig. 11c). At some intermediate value of L_1, the integral of the shear strain energy integrated over the length reaches a maximum value per unit length (Fig. 11b) and the relative energy dissipation is maximum.

The normal stress in the constraining layer and the shear stress in the viscoelastic layer for multiple layer treatment are found from Equation (28):

$$\sigma_2^* = E_2 \frac{\sigma_o}{2E_2} \left[1 + \frac{\sinh \frac{2x}{L_o^*}}{\sinh \frac{L_1}{2L_o^*}} \right] \tag{48}$$

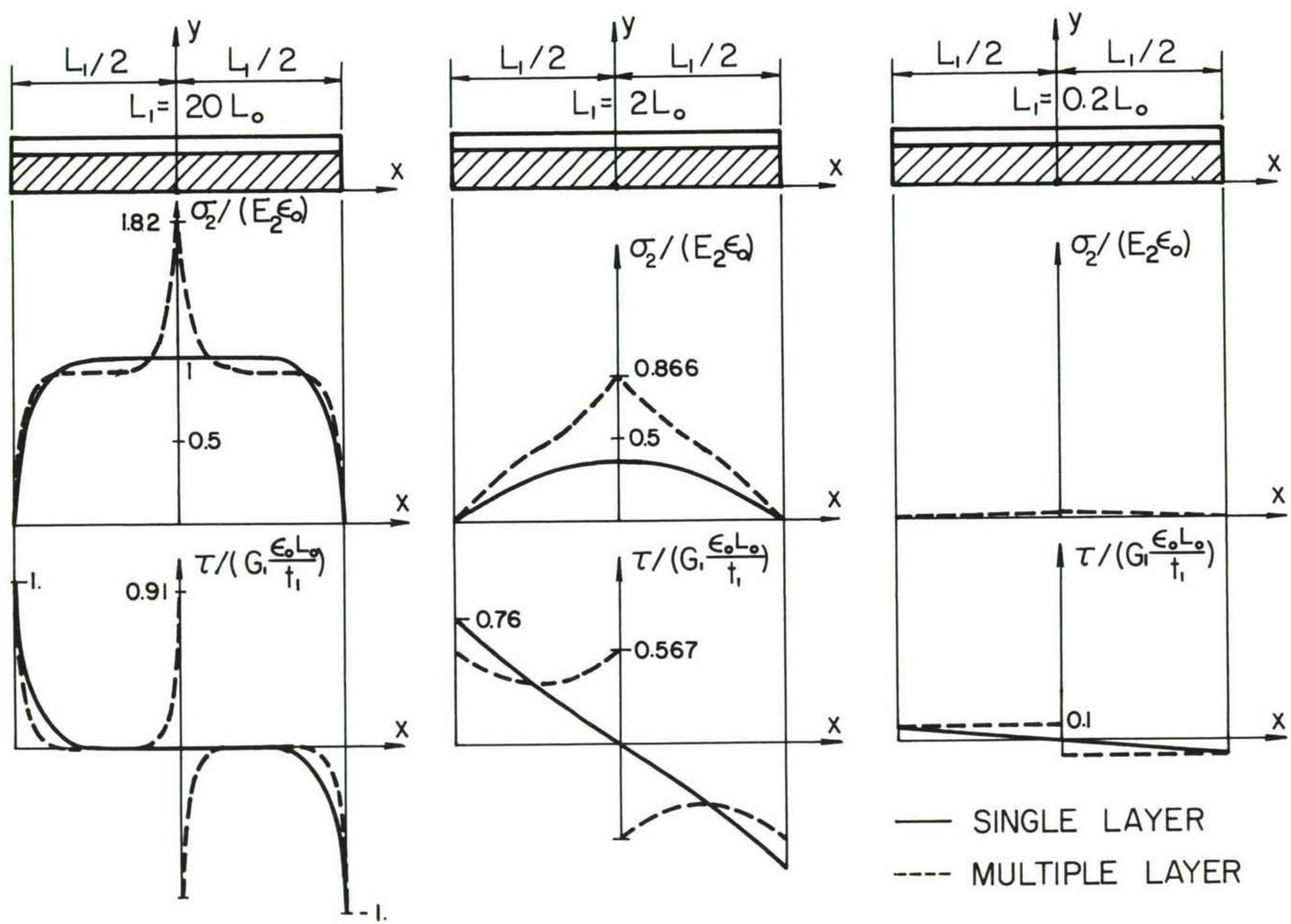


FIG. (11) STRESS DISTRIBUTION IN THE SURFACE TREATMENT

$$\tau^* = G_1^* \cdot \frac{\sigma_o}{2E_2} \cdot \frac{L_o^*}{t_1} \cdot \frac{\cosh \frac{2X}{L_o^*}}{\sinh \frac{L_1}{2L_o^*}} \tag{49}$$

To compare these results with those for single layers, we need the ratio between σ_o and ϵ_o, Equation (29):

$$\epsilon_o = \frac{\sigma_o}{2E_2}\left[1 + \frac{1}{\alpha^*} \cdot \coth \alpha^*\right]$$

where $\alpha^* = \frac{L_1}{2L_o^*}$

Equations (48) and (49) can then be written as

$$\frac{\sigma_2^*}{E_2 \epsilon_o} = \left[1 + \frac{1}{\alpha^*} \coth \alpha^*\right]^{-1} \cdot \left[1 + \frac{\sinh \frac{2X}{L_o^*}}{\sinh \alpha^*}\right] \tag{50}$$

$$\frac{\tau^*}{G_1^* \frac{\epsilon_o L_o^*}{t_1}} = \left[1 + \frac{1}{\alpha^*} \cdot \coth \alpha^*\right]^{-1} \cdot \frac{\cosh \frac{2X}{L_o^*}}{\sinh \frac{L_1}{2L_o^*}} \tag{51}$$

The existence of an optimum value of L_1 for maximum damping follows from the same argument as for the single layer treatment. Equations (50) and (51) are plotted in Fig. (11) for comparison with single layer treatment.

For a given constraining and viscoelastic layer, there is an optimum length for the elements of the constraining layer. While the optimum length increases indefinitely as the number of layers increases, there is a minimum value as the number of layers decreases. This minimum optimum length is about half of the optimum length for a single layer of exactly the same geometry and material because the assumptions of the multiple layer theory make the viscoelastic layer effectively stiffer. The geometry and boundary conditions of the two comparable composite configurations are shown in Fig. (12). For both problems, we have assumed that the strain is uniform at $y = 0$ which is the interface of the basic structure and the viscoelastic layer in the single layer case and is the middle surface of the viscoelastic layer in the multiple layer case. In the single layer analysis, $\tau = 0$ at $y = (t_1+t_2)$ and $\sigma = \sigma_o$ at $x = 0$, $y = t_1$.

In the multiple layer analysis, $\tau = 0$ at $y = \frac{1}{2}(t_1+t_2)$ and $\sigma = \sigma_o$ at $x = \frac{L_1}{4}$, $y = t_1/2$; the only other difference is that $u = 0$ at $x = 0$, $y = 0$ for the single layer but $u \neq 0$ at $x = \frac{L_1}{4}$, $y = 0$ for the multiple layer analysis.

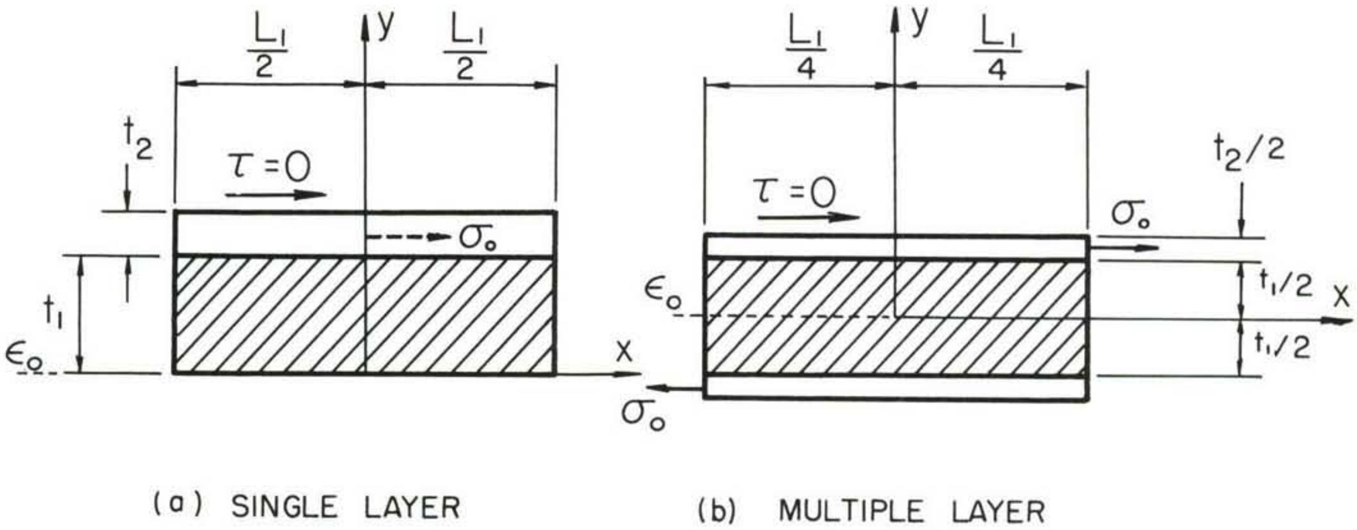


FIG.(12) GEOMETRY OF COMPARABLE BOUNDARY VALUE PROBLEMS

To make a legitimate comparison, it is necessary to replace t_1 by $2t_1$ and t_2 by $2t_2$ in the definition of L_o* to obtain the value of B_o* for the case of the single layer treatment. That is:

$$L_o = 2\left(t_1 t_2 \frac{E_2}{G_1}\right)^{\frac{1}{2}} = 2\,B_o$$

or

$$\omega = \frac{L_1}{B_o} = \frac{2L_1}{L_o} = 2\xi\,.$$

This is verified in part by the fact that the optimum element length for the single layer case is 3.28 B_o while for large n/m values in the multiple layer case it is about 1.7 L_o which is about the same physical length if the previous convention is used.

The value of ω for maximum damping in the case of single layer treatment is almost independent of the viscoelastic material used. In the case of multiple layer treatment, the value of ξ for maximum damping changes with the number of layers and so does the length of each element of the constraining layer. It can be seen from Fig.(18) that a given value of the dimensionless loss coefficient, η_1, for a given constraining and viscoelastic layer can be

obtained either with a small L_1 and few layers, m, or a large L_1 and many layers. The maximum value of η_1 for optimum ξ does not depend markedly on the number of layers which is given by the ratio m/n. Thus from Equation (42) the modified loss coefficient of the system, η_L, is almost linearly proportional to the number of layers if the element length is increased as the number of layers is increased. In actual applications, it is not practical to use different element lengths for different numbers of layers. If a fixed element length is used for multiple layer treatment, the amount of damping always increases with the number of layers but not necessarily proportionally. The predicted values of $(\eta_S - \eta_B)$ for $L_1 = 0.667$ inch and $L_o = 0.236$ inch are shown in Fig. (21) where $(\eta_S - \eta_B)$ is almost exactly proportional to m for m = 1 to 15. This linear relation will not hold for large m because η_1 decreases for large m for this particular geometry as shown in Fig. (13).

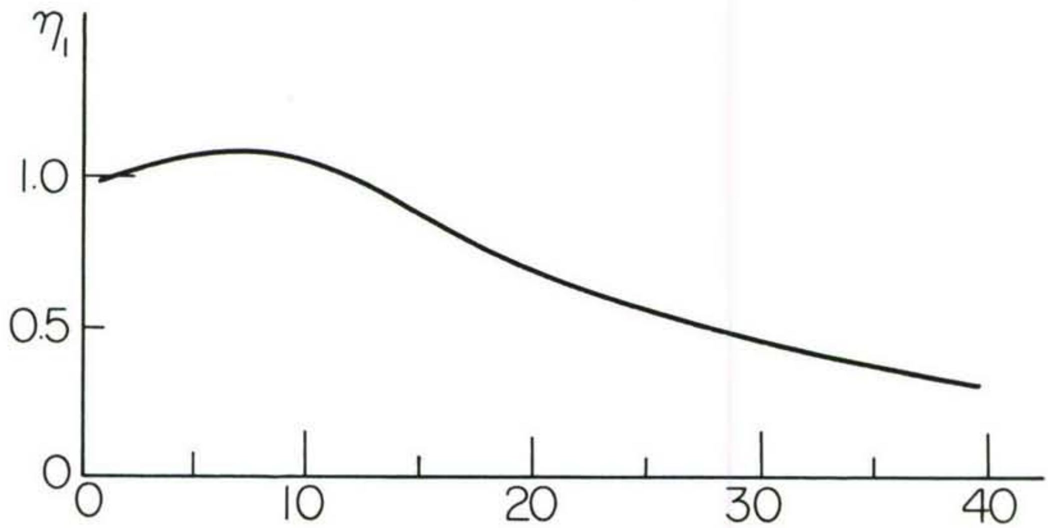


FIG. (13) η_1 AS A FUNCTION OF m

The multiple layer theory is valid if the element length of the constraining layer is much shorter than the total length of the surface treatment. If L_1 approaches L, the strain at the middle surface of the viscoelastic layer will not be uniform and assumptions used in the derivation of the equivalent homogeneous material will be violated and the damping will be overestimated.

In Equations (18) and (42) the shear modulus of the viscoelastic material does not appear explicitly; the loss coefficient of the system depends primarily upon the stiffness of the constraining layer and the strain energy of the basic structure and only

indirectly on the shear modulus of the viscoelastic layer through the element length ratio L_1/L_o. Viscoelastic materials with high loss factors like 3M No. 466, usually have a shear modulus which is very temperature dependent. If the element length, L_1, is chosen to be optimum for the center of the temperature range, the system damping can be designed to be almost constant over a large temperature range in spite of this. For example, if L_1 is chosen so as to make L_1/L_o optimum at 65°F, η_1 is still as great as one half of its maximum value at 30°F and 110°F even though the shear modulus changes by a factor of 30 to 1 over this same range. Fig. (14) shows η_1 as a function of temperature for 3M No. 466 with G_1 obtained from the master curve for f = 72 cps.

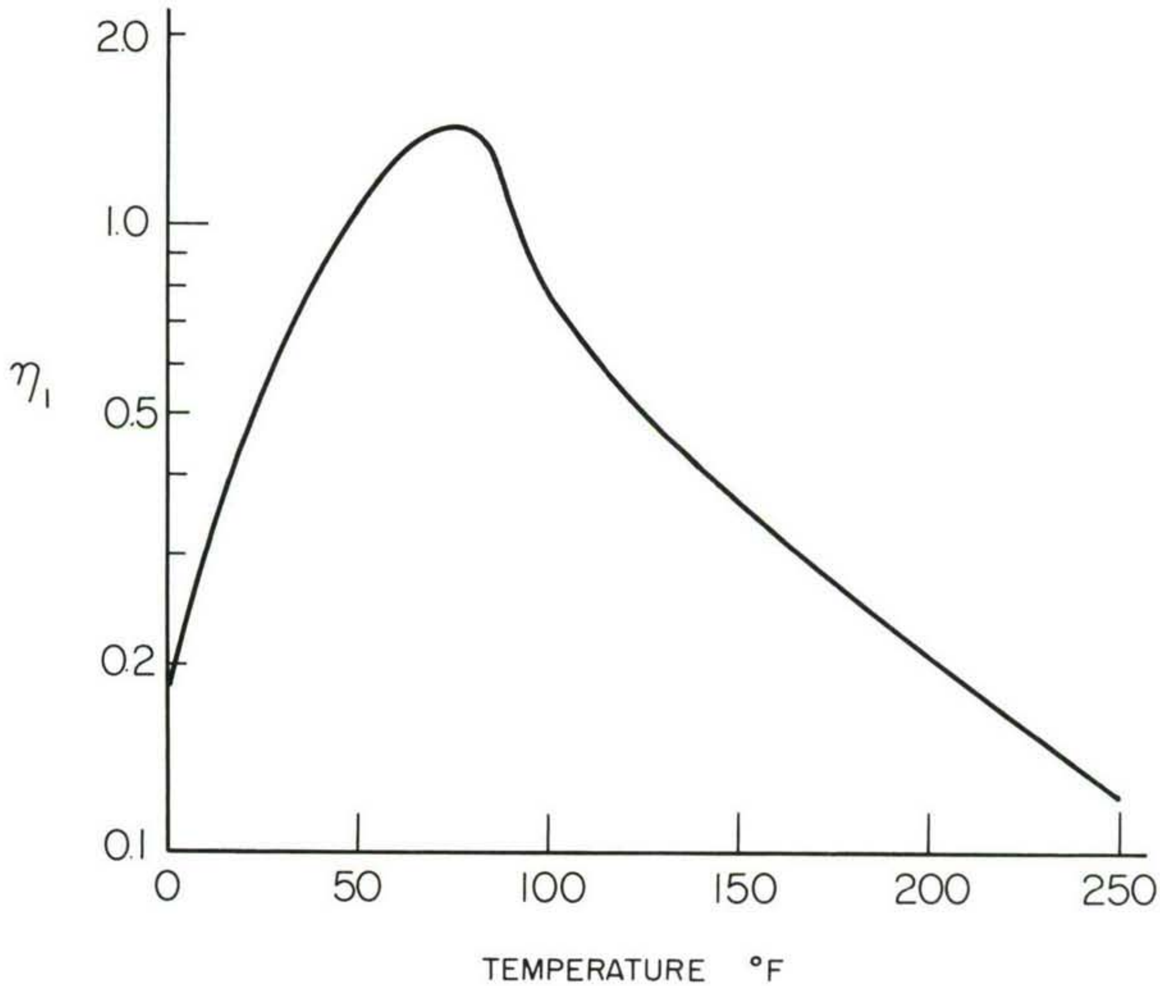


FIG. (14) η_1 AS A FUNCTION OF TEMPERATURE.

VI

CONCLUSIONS

The experimental results given in this report agree with the damping predicted for constrained viscoelastic layers based upon the assumption that the energy dissipation is caused primarily by the shear strain in the viscoelastic layer. The effective elastic modulus method used in the analysis of a multiple layer treatment proved to be satisfactory for the study of laminated structures. One important result found from the analysis is that, for optimum element length of the constraining layer, the energy dissipation depends primarily upon the loss coefficient of the viscoelastic material, and the stiffness of the contraining layer and only indirectly on the shear modulus of the viscoelastic layer.

References

1. Kerwin, E. M. Jr., "Damping of Flexural Waves by a Constrained Viscoelastic Layer" J. Acoustic So. of Amer. Vol. 31., pp 952-962, July 1959.

2. Parfitt, G. G., "The Effect of Cuts in Damping Tapes", 4th International Congress on Acoustics, Copenhagen, pp 21-28, August 1962.

3. Lazan, B. J., Metherell, A. F., and Sokol, G., "Multiple-Band Surface Treatments for High Damping", AFML-TR-65-269, September 1965.

4. Oberst, H., "Uber Die Dampfung Der Biegeschwingungen Dunner Bleche Durch Fest Haftende Belage", Acustica 2, Akustische Beihefte 4, pp 181-194, 1952.

5. Kerwin, E. M. Jr., "Macromechanism of Damping in Composite Structures", ASTM, 1965.

6. Ross, Donald, Ungar, Eric E., Kerwin, E. M. Jr., "Damping of Plate Flexural Vibrations by Means of Viscoelastic Laminae", Structural Damping, pp. 49 - 87, ASME, 1959.

7. Lazan, B. J., Damping of Materials and Members in Structural Mechanics, Pergamon Press, 1968.

8. Bishop, R. E. D. and Johnson, D. C., The Mechanics of Vibration Cambridge, 1960.

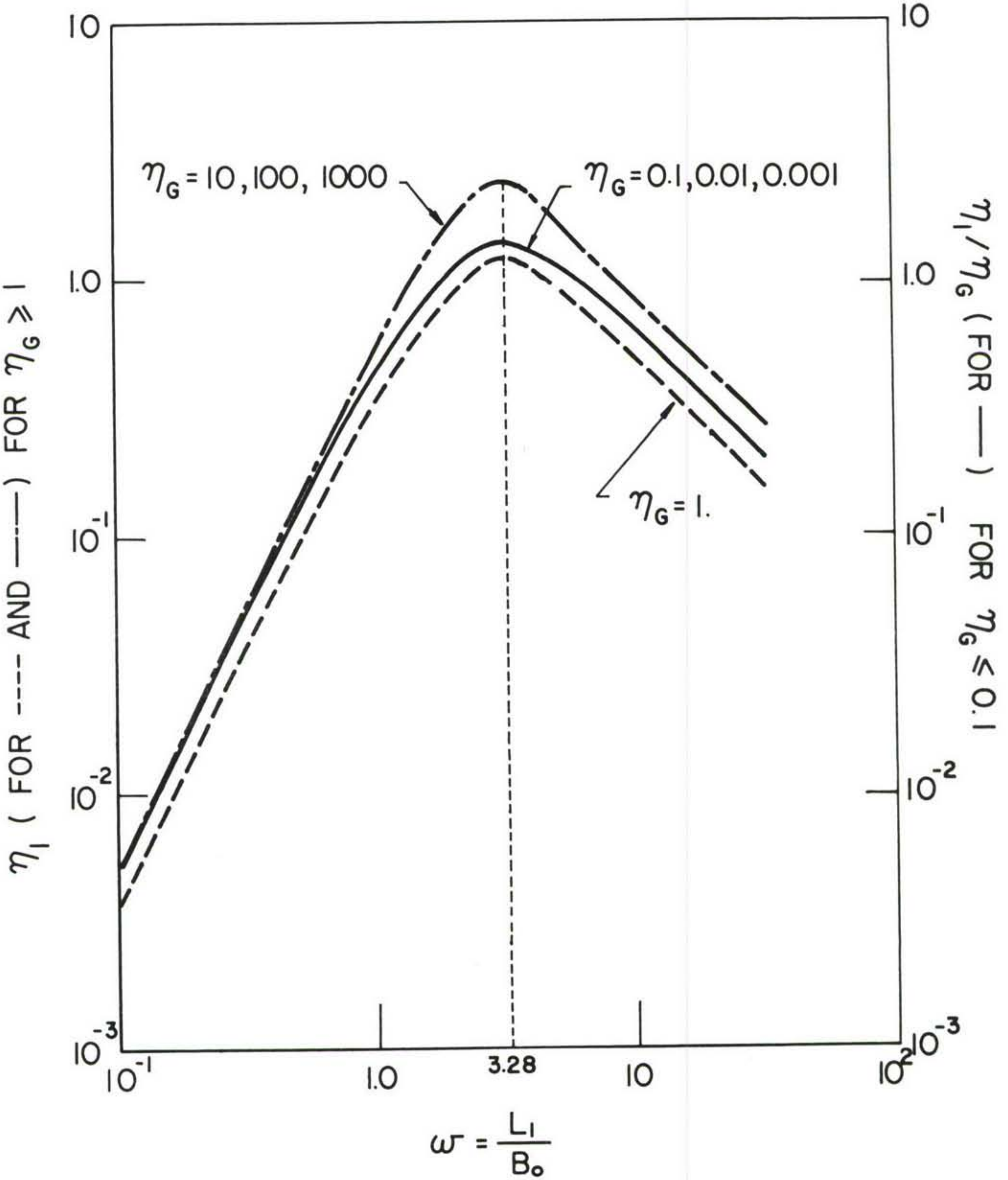


FIG. (15) η_1 VERSUS $\omega = L_1 / B_o$ FOR SINGLE CONSTRAINING LAYER.

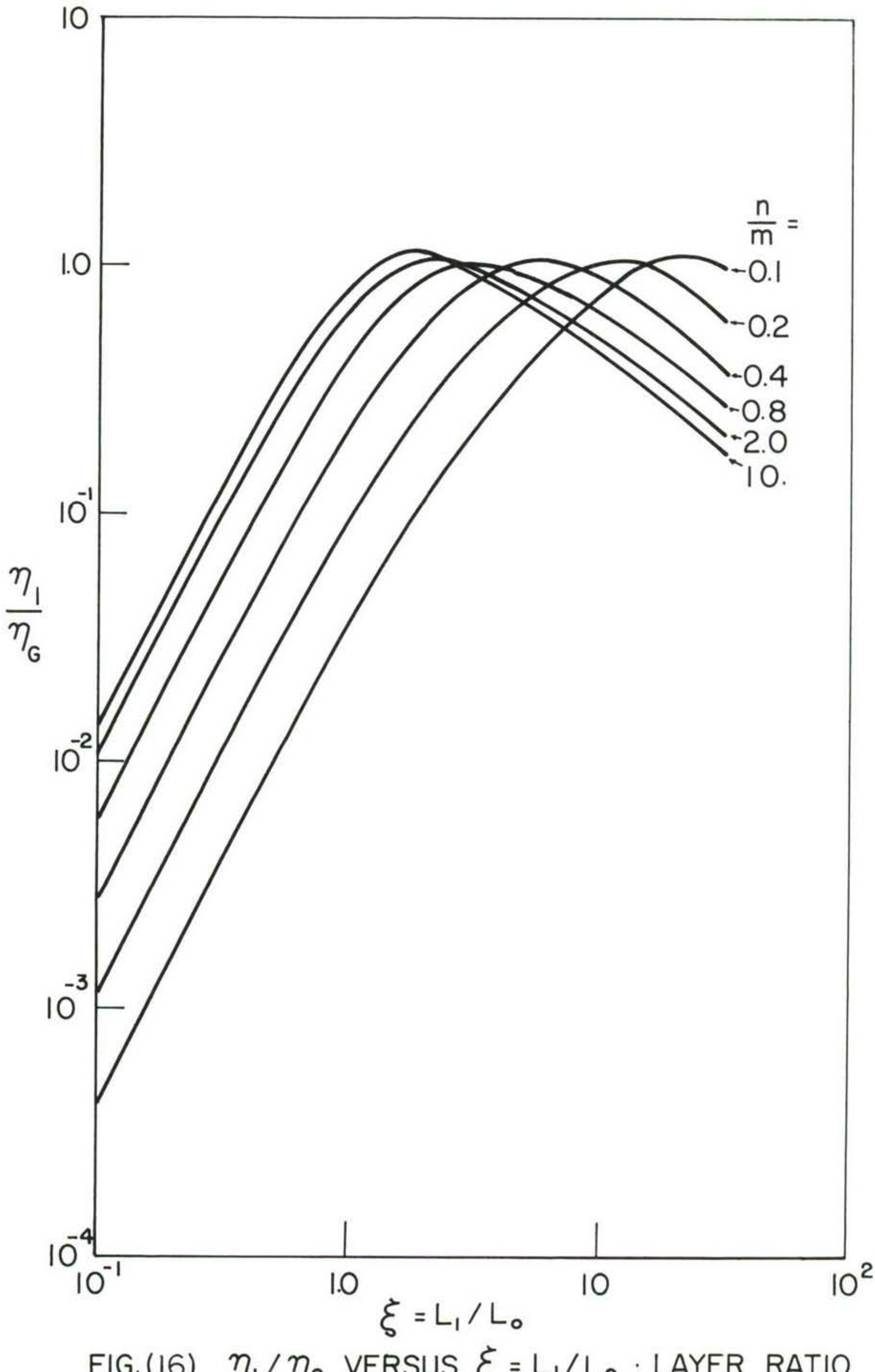


FIG. (16) η_1/η_G VERSUS $\xi = L_1/L_0$; LAYER RATIO n/m AS A PARAMETER. η_G=0.1, 0.01, 0.001.

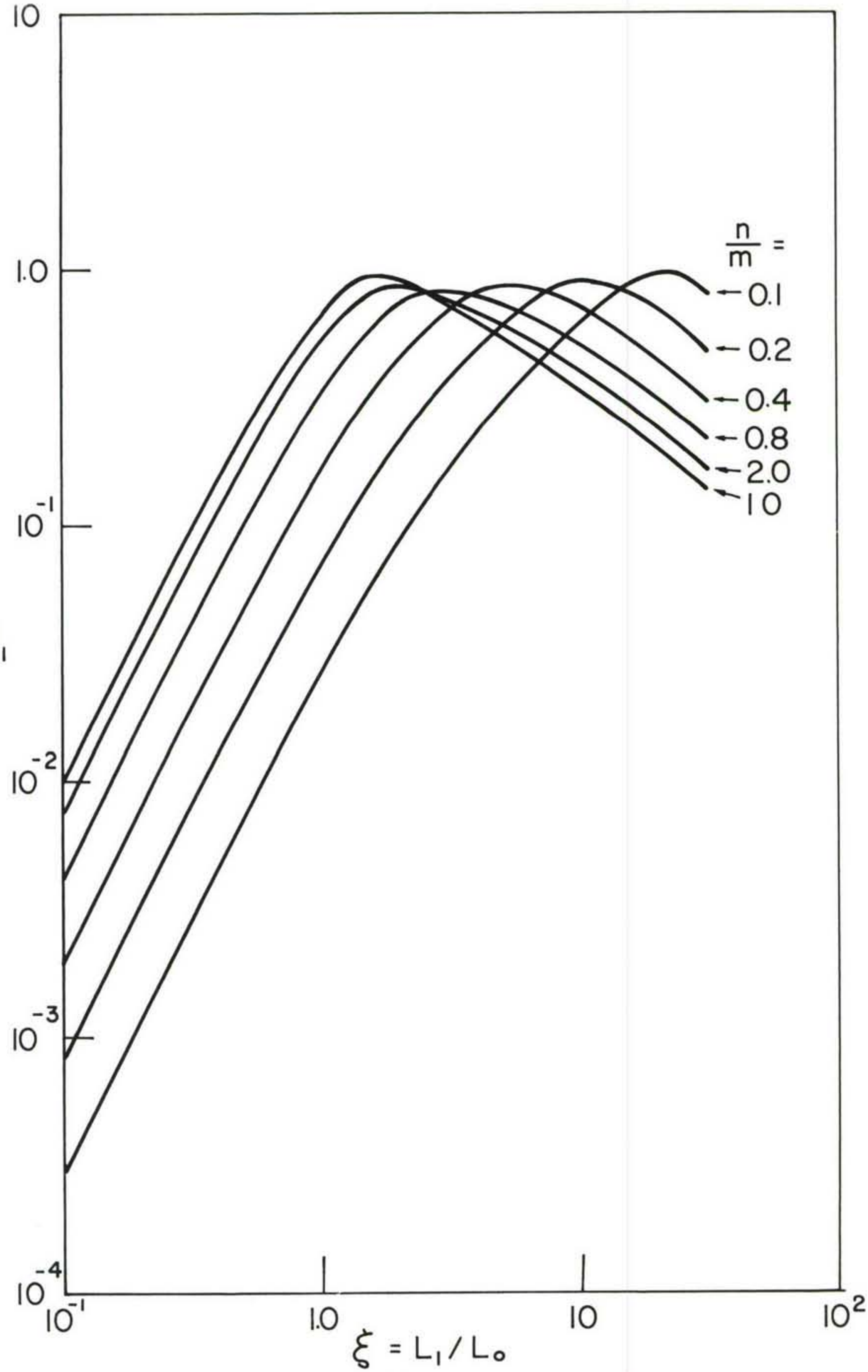


FIG. (17) η_1 VERSUS $\xi = L_1/L_o$; LAYER RATIO n/m AS A PARAMETER. $\eta_G = 1.0$.

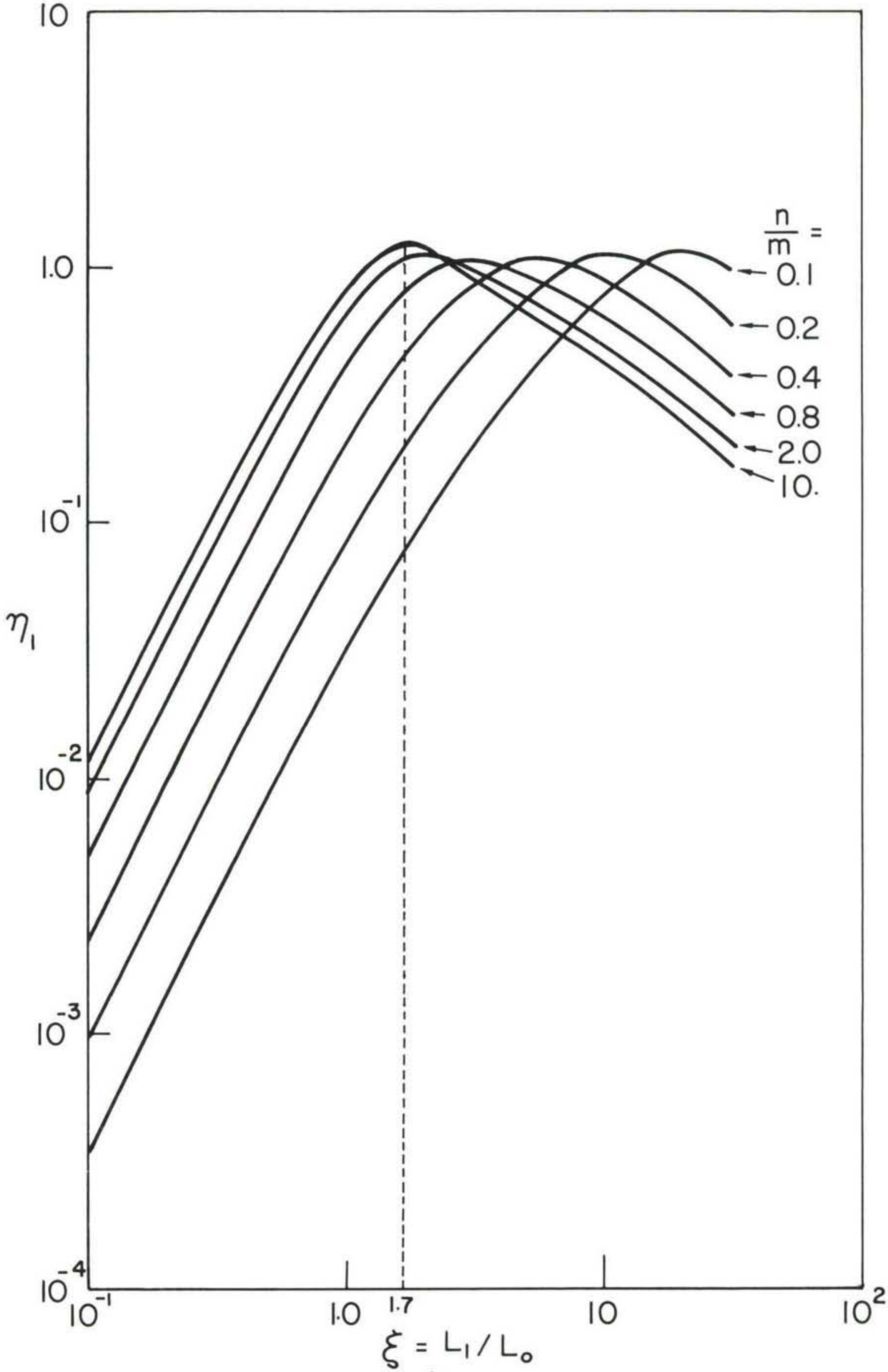


FIG. (18) η_1 VERSUS $\xi = L_1/L_0$; LAYER RATIO n/m AS A PARAMETER. $\eta_G = 1.5$.

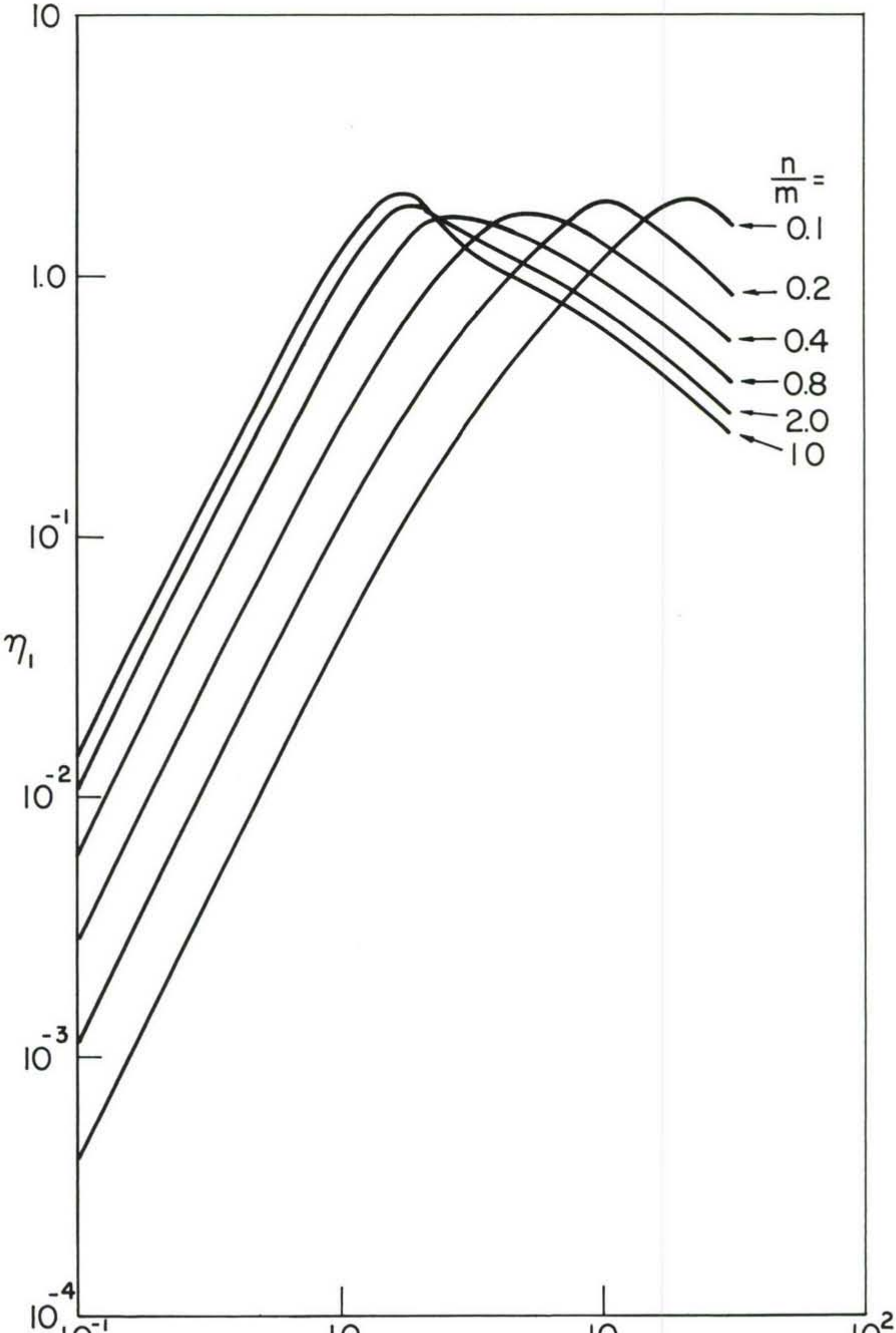


FIG. (19) η_1 VERSUS $\xi = L_1/L_0$; LAYER RATIO n/m AS A PARAMETER. η_G = 10, 100, 1000.

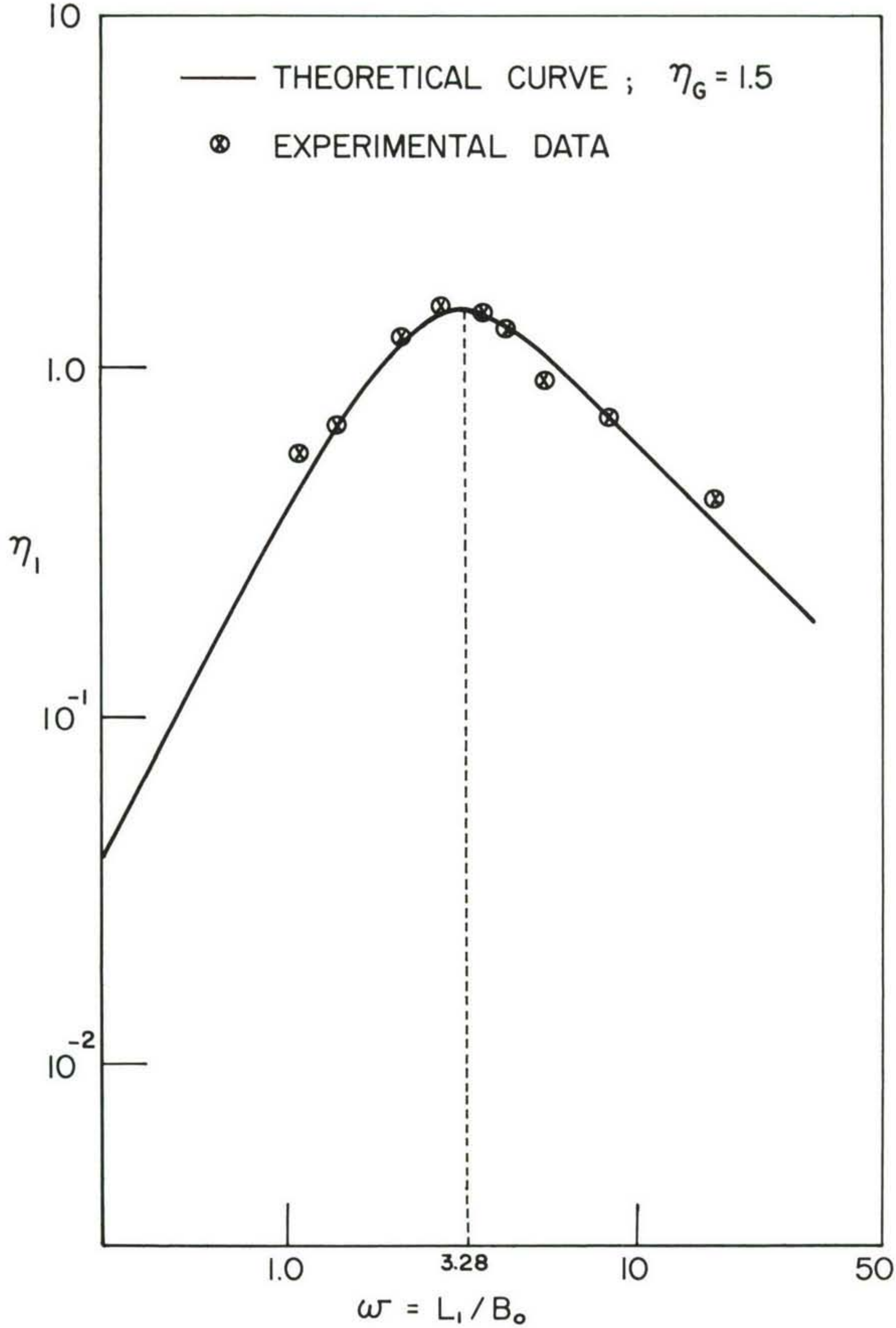


FIG. (20) COMPARISON OF THEORETICAL PREDICTIONS AND EXPERIMENTAL RESULTS FOR SINGLE CONSTRAINING LAYER.

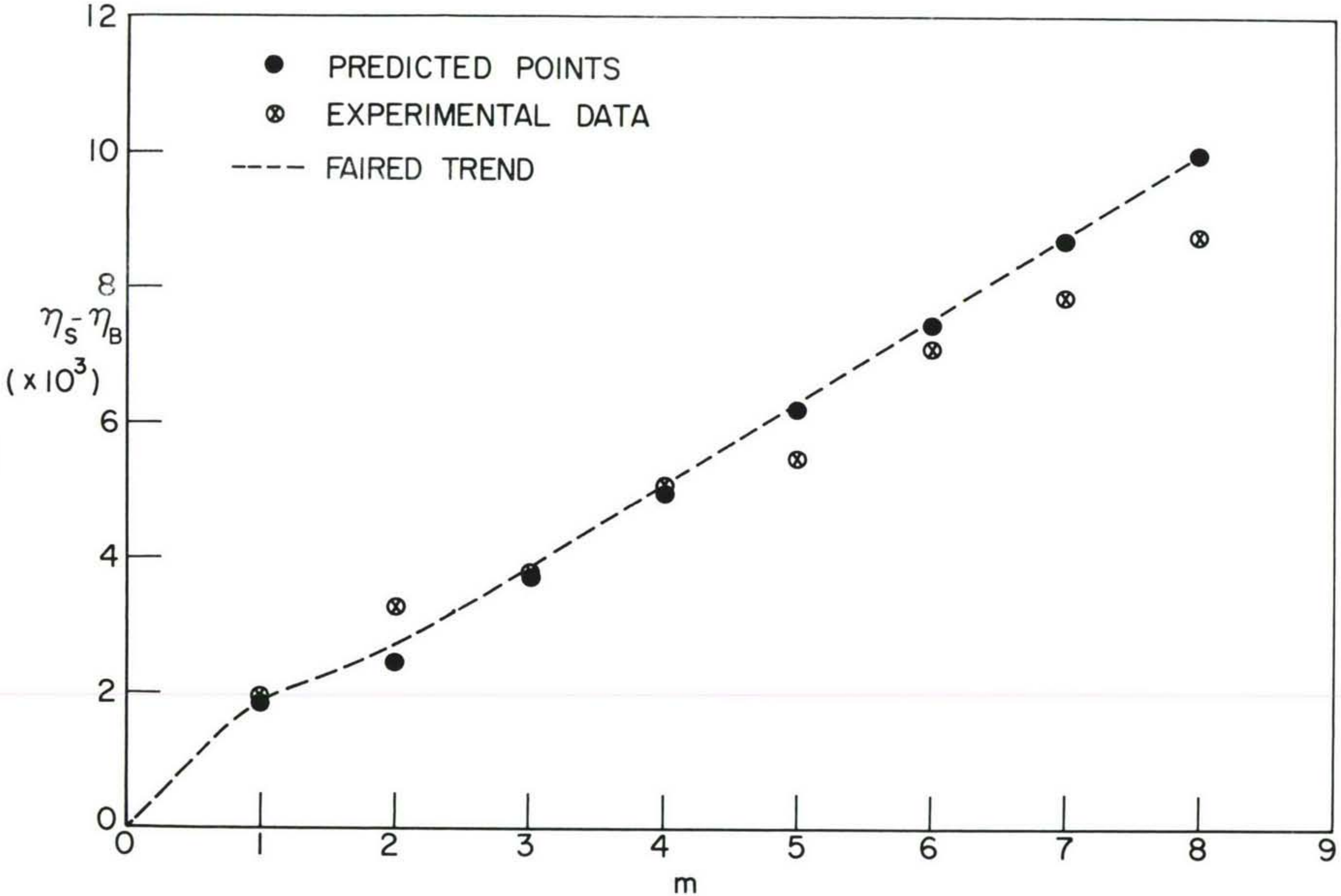


FIG. (21) COMPARISON OF THEORETICAL PREDICTIONS AND EXPERIMENTAL RESULTS FOR MULTIPLE CONSTRAINING LAYERS.

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) University of Minnesota Minneapolis, Minnesota 55455	2a. REPORT SECURITY CLASSIFICATION unclassified 2b. GROUP

3. REPORT TITLE

LENGTH OPTIMIZATION FOR CONSTRAINED VISCOELASTIC LAYER DAMPING.

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final Report - May 1967 through October 1968

5. AUTHOR(S) (Last name, first name, initial)

R. Plunkett and C. T. Lee

6. REPORT DATE July 1969	7a. TOTAL NO. OF PAGES 55	7b. NO. OF REFS 8

8a. CONTRACT OR GRANT NO. F33615-67-C-1667 b. PROJECT NO. 7351 c. 735106 d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) AFML-TR-68-376

10. AVAILABILITY/LIMITATION NOTICES This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Metals and Ceramics Division (MAM), Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio 45433.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY AF Materials Laboratory (MAMD) Air Force Systems Command Wright-Patterson AFB, Ohio

13. ABSTRACT

Viscoelastic materials are extensively used to damp flexural vibrations of metallic structures; it has been known for some time that the energy dissipation due to shear strain in the viscoelastic layer can be increased by constraining it with a stiffer covering layer. In this report we will discuss a method for increasing this damping by cutting the constraining layer into appropriate lengths. The analysis for a single layer of this treatment is relatively straightforward. The damping can be increased still further by using several layers; in this case the analysis is based upon effective complex elastic moduli of an equivalent homogeneous medium. One result found from this analysis is that, for optimum spacing of cuts, the damping depends primarily upon the stiffness of the constraining layer and only slightly on the shear modulus of the viscoelastic layer. Experimental data is presented for comparison with the theoretical predictions.

This abstract is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Metals and Ceramics Division (MAM), Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio 45433.

DD FORM 1 JAN 64 1473

unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through
_______________________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through
_______________________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through
_______________________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.